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BENJAMIN FRANKLIN'S "MAGICAL SQUARE OF 16"

BY

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A great printer must turn in his grave while typographical errors make him appear slipshod, even in mathematics where he commanded a wide reputation. Eight such misprints on one page of the ten-volume *The Life and Writings of Benjamin Franklin*, Macmillan, 1905, would indicate that Franklin could not count up to 256.

Facing page 458 of Vol. 2 is the "Magical Square of 16" sent by Franklin to Peter Collinson, an English scientist. But, as printed, the figure bristles with departures from Franklin's letter which states that each column, line and diagonal adds up to 2056 as required in such a square. The numbers 20 and 181 and 241 and 251 appear twice each. Franklin did not stutter in dictating to a stenographer, but wrote with his own hand. He must have been proud of the result, to have sent it to Collinson, who must have been satisfied too—else he would not have presented the matter to the Royal Society, to which Franklin was admitted as a member.

To make room for these duplications, the printer omitted four other numbers, 185 and 203 and 211 and 255. Of course some of the totals could not be what Franklin said they were. (See Fig. 1.)

The vice of misprinting Franklin has come across the water. Similar misprints in *Magic Squares and Cubes*, by William S. Andrews, Chicago, 1906, duplicate the numbers 55 and 90 and 241, and omit 53 and 99 and 211 from this famous square over which there has been nearly two hundred years of puzzling by men inclined to "mathematical recreations."

The man of common sense who said "Love your neighbor but don't pull down your hedges" would have seen to it that his diagonals added to 2056 before submitting the computation to the scrutiny of some half dozen of his scientific friends. He could add. What he submitted to

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them was a composition of elaborate harmonies and antitheses with no more likeness to a simple magic square than a fugue has to a song.

Franklin's magic square, as described in his letter, can be reconstructed. We will discern something of his method if we begin by ascertaining which of these duplicate numbers is to remain and which is to be stricken; then how each blank is to be filled by one of the omitted numbers. This can be done with absolute certainty.

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
20	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
203	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	251	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

FIG. 1. Franklin's Magical Square as reproduced in "The Life and Writings of Benjamin Franklin," by Albert Henry Smyth, Vol. 2, New York, The Macmillan Company, 1905.

As one looks at a checkerboard it is obvious here that Franklin's great square is a mosaic, made up of tiny squares, of four numbers each. Every square group of four numbers adds up to 514. For example, in the top left corner are the numbers 200, 217, 39 and 58 in a square—total 514. Everywhere, left, right, and in the middle, this perfectly woven

fabric is the base upon which the design is worked. To complete a tiny square, from which any duplicated number is stricken, only one of the omitted numbers can be fitted in and fill out a tiny square which adds up to 514.

With these typographical corrections, to begin with, the magic square so carelessly published will reappear as shown in Fig. 2.

Yet discrepancies remain between this partly restored square and Franklin's description of it. Both diagonals are wrong. Where did

200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	185
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65

FIG. 2.

the published square come from in the first place? It was printed in London, as a supplement to a letter from Franklin to Collinson, in a quarto volume entitled "Experiments and Observations on Electricity, made at Philadelphia in America, by Benjamin Franklin LL.D. and F.R.S. To which are added Letters and Papers on Philosophical Subjects. David Henry, 1769." That was after Collinson's death; and Franklin learned of the publication from his son. The letter bore no date—thus differing from Franklin's habit. Neither the original nor Franklin's copy has been found. Throughout years of regular correspondence Franklin and Collinson habitually referred back to former

letters by their dates. The omission of a date, in the letter as published, raises doubts of the extent of its integrity in other particulars. Upon that piratical publication, later mathematicians have objected that the diagonals do not add as required, and as stated by Franklin. Obviously one makes 1928 and the other 2184; one 128 too small, and the other 128 too large. So the whole is not a "magic square" at all.

But this shows only that the production, in that form, is unfinished. By a routine step in completing many such squares, that fault can be corrected in a minute or two merely by reversing the lower half. It is as simple as that. Then both these diagonals will add up to 2056. The lower half of column No. 1 becomes the lower half of column No. 16, and vice versa; the lower halves of columns 2 and 15 change places, and so on.

If what we have be not a preliminary draft, the fault could occur in another way. In setting type by hand, each column of figures being held together by strips of lead on either side, a common mistake was to start on the wrong side in transferring columns from the "stick" in the printer's hand to the form for the press. Similarly a line of children may take places with the foot of the class standing where the head usually stands. So printers can shift numbers inadvertently, and Benjamin Franklin marched them about like soldiers in squads and companies; in his great square he even undertook to make the four figures at the corners equal the four figures at the center. And they do.

To discern his manipulation from the very start, look at the lowest row in this great square. The smallest figure, 1, is at the bottom of the fourth column; next to it is the largest figure, 256. Above these stand 2 and (as corrected) 255; above these 3 and 254 are paired. The series from 1 up to 16, and the series from 256 down to 241, are maneuvered in pairs, within the squads of four mentioned above, along one vertical. No single number is used at random. Each balanced pair adds up to 257.

How did such a mind as Franklin's come to be occupied with these *difficiles nugae*, as he called them, fruitless puzzles? It was while Clerk of the Pennsylvania Colonial Assembly, or legislature, before he became a member of that body. During dull debates he beguiled the time making such squares "and at length had acquired such a knack at it that I could fill the cells of any magic square, of reasonable size, with a series of numbers as fast as I could write them." What a hint from Poor Richard to members of the Gridiron Club who must listen to Congressional proceedings!

These efforts culminated in his creating the now fragmentary Magic Square of 16, which he claimed to be "the most magically magical of any magic square ever made by any magician." Even today, according to Andrews, mathematicians "do not know Franklin's method." The

mathematician James Ferguson received a copy with permission to print it and declared it "far beyond anything of the kind I ever saw before."

Franklin sent another copy of this square to John Canton before May 29, 1765. As late as 1772 he may not have seen the 1769 publication. For on December 26, 1772, he wrote Dr. Barbeu Dubourg in Paris thanking him for a square and saying, "I had somewhere a square of 8 with the diagonals you required, which I would send you if I could find it." Such a square of 8 was printed with the square of 16 in the 1769 publication, and reprinted in French and German translations. Why copy longhand or ransack his papers for printed matter available in Paris? Or had he seen and chosen to ignore that publication? In a letter to Ilgenhousz he said, of another controversy, "an answer was expected from me, but I made none to that book, nor to any other. . . . You can always employ your time better than in polemics."

From the fragments we have, with all their resemblance to material in process, a magic square of 16 can be completed to fit Benjamin Franklin's description, with the same assurance with which missing numbers were restored above. Some variations in detail are possible, because Franklin's letter mentions his square of 8 as a model without making clear how far the resemblance is to go.

The scheme of Franklin's Magic Square of 16 is a mat of four broad zones, of four horizontal lines each; crossed by four broad tiers, of four vertical columns each. This creates a checkerboard of sixteen large squares, four in each zone and four in each tier. For the sake of clarity these are separated somewhat in the restoration below. And for convenient reference, as we go along, let them be designated as:

A B C D
E F G H
I J K L
M N O P—in that arrangement.

Now look at A at the left of the uppermost zone. In its first left column the four numbers add up to 516; in the next column the four numbers make 512; and so the columns alternate all across this uppermost zone, 516, 512, 516, 512. But that order is reversed in the zone next below, the footings in E, F, G, and H of the pattern being 512 first, then 516, and so on. In this particular the third zone is like the first, and the fourth like the second. That alternation must be retained; because putting side by side any two little columns, which add up to 516 each, would make it possible to find an internal square of 16 cells which do not add up to 2056. But Franklin declared that every such mosaic group of 16 cells would make 2056, and dwelt upon that fact as a peculiarity and novelty of his great square.

The reconstructor's hands are tied by another of his specifications: each half-diagonal, and its parallels, must equal every other half-

diagonal. The only answer to that is to make all equal 1028. That had not been accomplished by simply reversing the lower half of the whole square, the familiar device suggested above for making the whole diagonals add up to 2056 each, if that alone were required.

When our family circle finds fault with the papering of a room, they do not blame the manufacturer, but complain that the paper-hanger has not fitted the edges of the rolls together as the pattern requires. Similarly we shall find the trouble with Franklin's square of 16, as published, is at the seams between what we have called zones and tiers. There is nothing wrong with the mosaic fabric of tiny squares of four numbers each adding up to 514, upon which the pattern is imposed, nor anything inherently wrong with the sixteen larger mosaics lettered A to P above. The trouble is that a preliminary layout has been published, not the final assembly which could satisfy either Franklin or his correspondents.

If Franklin sent his friends only parts of successive steps in perfecting his design, a few numbers would sometimes suffice. For example, in the square of 4 below, to give only five numbers at any corner will enable a mathematician to complete the whole; so between skilled correspondents why write out eleven other numbers longhand? But nothing could be less intelligible to a printer or more prolific of errata if less than the whole series were printed to save space or expense, and without submitting proofs to the author.

To lift the veil of obscurity, which printers have draped over Franklin's achievement, calls for mathematical detective work—and first a paragraph to demonstrate that numbers, however rigid alone, are subject to manipulation in groups. For example, magic squares of three and four can be made this way:

THREE		
8	1	6
3	5	7
4	9	2

Total 15, up, across or askew.

FOUR			
6	9	12	7
3	16	13	2
15	4	1	14
10	5	8	11

Total 34.

As three times four make twelve, and also four times three make twelve, a magic square of twelve can be built from this material. Repeat three-squares in place of each number in the four-pattern, or repeat four-squares in place of each number in the three-pattern. That is, for number 1 in the square of three above, substitute 1 to 16 in the pattern of the square of four. Then proceed with 17 to 32 in place of the figure 2. Instead of number 3 go on with 33 to 48; and so on. The square of twelve finally, using numbers 1 to 144, will add to 870 vertically, or horizontally or diagonally. And the same totals will be reached if squares of three are projected in the pattern of the square of four. In either case the result will be technically a "perfect even square" made

up of pairs of numbers, each two numbers skewly (to use an old word saved by mathematicians) opposite, and equidistant from the center, making 145. Thus 1 and 144, 45 and 100, and 70 and 75 are balanced in both the squares.

If we pick to pieces Franklin's larger mosaics, those squares of sixteen numbers each, A and B and C and D of our pattern, in the topmost zone,

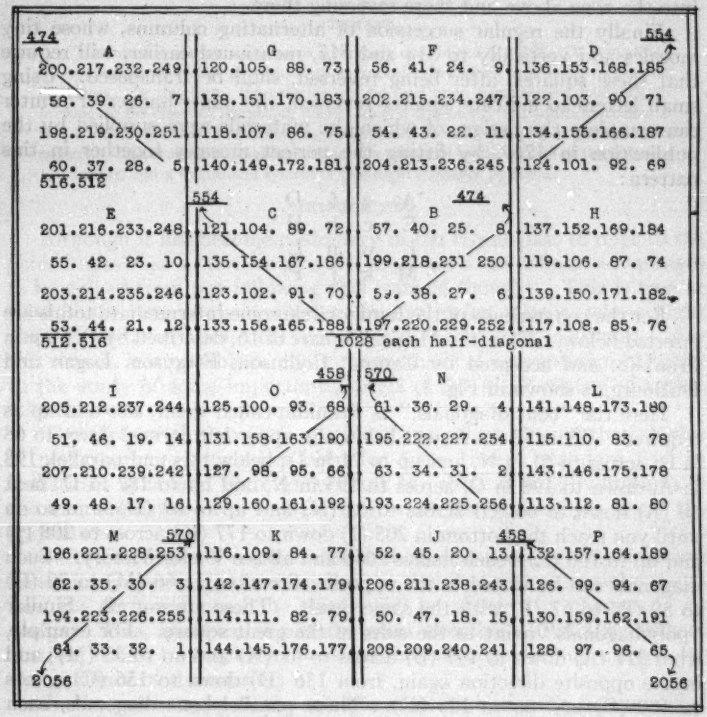

FIG. 3.

all add diagonally to 474 from upper left to lower right corner; but each adds to 554 diagonally from upper right to lower left. Diagonals in all the zones below make entirely different sums. Stated in the same order, those in squares E and F and G and H make 490 and 538; in the third zone, 506 and 522; and in the lowest zone, 570 and 458.

To perfect the diagonal in A at the uppermost left corner, which adds to 474, we find only one of all these numbers can be added to it to

make the required 1028; that is 554, found only in the opposite diagonals in the same top zone. Obviously one of these squares, B or C or D, must be dropped down into the zone next below, and reversed, to project 474 through 554 and produce 1028.

Similarly in the lower half of the great square, the diagonals in the corners M and P can be made to produce 1028 only by lifting N and O into the zone above and there reversing them.

Finally the regular succession of alternating columns, whose tiny mosaics add vertically to 516 and 512, mentioned earlier, will require that these squares, after being reversed, shall be transposed. Using small letters to indicate squares reversed, the paper-hanger or printer can complete the design, of which the materials were supplied by the publication in 1769, by fitting the perfect mosaics together in this pattern:

A	g	f	D
E	c	b	H
I	o	n	L
M	k	j	P

For the convenience of the family circle some intermediate totals are inserted below. Thus the Magical Square of 16, described by Benjamin Franklin, and accepted by Canton, Collinson, Ferguson, Logan and DuBourg, is shown in Fig. 3.

Here the "bent diagonals," as Franklin called them, also add up as required. Thus 58 (in A next below the upper left corner) down to 68 in O, across to 61 in N, and up to 71 in D; below this and parallel, 198 in A, down to 190 in O, across to 195 in N, and up to 187 in D; next 60 (A) down to 66 (O) across to 63 (N) and up to 69 (D); and so on until you reach the bottom in 205 (I) down to 177 (K) across to 208 (J) and up to 180 (L)—each makes 2056 and all are V's for Victory. Such diagonals can be traced in the opposite direction, as 66 (M) to 70 (C) to 59 (B) to 67 (P) with the same result. These are not all. Similar "polydiagonals" point to the sides of the great square. For example, from 217 (A) down to 197 (B) across to 61 (N) and on to 33 (M); and in the opposite direction again, from 136 (D) down to 156 (C) across to 100 (O) and on to 128 (D). These parallel bent diagonals, each making 2056, laced through the mosaic fabric of tiny and intermediate squares, weave "the most magically magical of any magic square ever made by any magician."

THE PERTURBATION CALCULUS IN MISSILE BALLISTICS

BY

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SUMMARY

The calculus of perturbations, as commonly used in exterior ballistics, is modified for application to rocket missiles, that is, to projectiles which experience thrust and lift forces. In this form, the calculus can be used as in conventional ballistics, to investigate the effect of disturbing factors upon a trajectory. In addition to this, it is of considerable value in establishing analytically certain principles of guidance and control. Two examples are given of this: the theory of range control of the German V-2 missile, and a theoretical system of free flight guidance by radio.

INTRODUCTION

Although it has become customary in the recent past to refer to the guided missile as a pilotless aircraft, there are some definite advantages to be gained from not following this train of thought. This is true at least in the case of the ground-to-ground rocket missile, whose flight resembles that of a projectile more than it does that of an aircraft. For this reason, it is possible to apply the methods of conventional ballistics to the study of some important aspects of the flight control of rocket missiles.

The topic of this paper is specifically an adaptation of the calculus of perturbations, which has been for some time a standard tool in ballistics. It is used there to compute the effect of small variations in the characteristics of a shell, a gun or of the atmosphere upon the trajectory.

In the ballistics of rocket missiles the calculus of perturbations can be similarly used. It then supplies the designer with essential data on how the performance of the missile will be affected by changes in its design. This, however, is not the most effective use of the method. Rather, it can be employed to derive important rules and criteria for the guidance of the missile.

In the present paper, the calculus of perturbations is first modified from the form it takes in conventional ballistics so that it applies to controllable missiles. It is then shown that it can be used to derive a criterion for the correct timing of the power cutoff. A criterion of this nature was the basis for the range control of the German V-2 rocket. Finally, an example is given of how the same general method will yield a guidance concept for the free flight portion of the trajectory. It is assumed there that the missile is remotely controlled from one or more radio stations.

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The treatment is strictly theoretical in all cases and no effort is made in this paper to take account of many of the practical difficulties which might be encountered in realizing the solutions. The purpose of this study is not to present eminently practicable control principles but to show the usefulness of the analytical method.

CALCULATION OF THE BASIC TRAJECTORY

In order to calculate a missile trajectory one must integrate a system of differential equations of the fourth order, which is often written in the form:

$$\begin{aligned} \dot{x} &= v_x, \\ \dot{y} &= v_y, \\ \dot{v}_x &= \phi_T(v_x \cos\alpha + v_y \sin\alpha) - v_x\Delta + v_y\Lambda, \\ \dot{v}_y &= -\phi_T(v_x \sin\alpha - v_y \cos\alpha) - v_y\Delta - v_x\Lambda - g, \end{aligned} \tag{1}$$

where ϕ_T, Δ, and Λ are related to thrust F_T, drag D, and lift L, of the missile by

$$\phi_T = \frac{F_T g}{Wv}, \qquad \Delta = \frac{Dg}{Wv}, \qquad \Lambda = \frac{Lg}{Wv}, \qquad (v^2 = v_x^2 + v_y^2).$$

The angle of attack α which enters these equations in the thrust terms as well as implicitly through the drag and lift coefficients C_D and C_L is an undetermined parameter. Its appearance in the equations of motion (1) constitutes the chief departure of guided missile ballistics from conventional ballistics. For, within certain limits, α can be subjected to arbitrary conditions and the flight path of the missile can be shaped accordingly to fulfill desired specifications.

There is one additional parameter in these equations which can be disposed of rather freely and which, therefore, offers the possibility of some control over the progress of the flight. It is the burning time, that is, the time during which the thrust does not vanish. In the case of the German V-2, for instance, range control was effected by a proper adjustment of the burning time.

It is true that this description does not exhaust the possibilities of flight control. It does, however, cover the methods which have so far been studied thoroughly or realized in practice.

It should, perhaps, also be pointed out that Eqs. 1 are derived in this form under certain simplifying assumptions which treat the missile as a mass point without inertia, and neglect the curvature and rotation of the earth.

The integration of system (1) is usually carried out numerically or with the help of mathematical machines. Before it is undertaken, dispositions must have been made concerning the two adjustable parameters: burning time and angle of attack. The end of burning is usually effected on the basis of some requirement which will be referred to as

the "cutoff criterion." The angle of attack is determined as a function of time by subjecting the flight to some additional specification. This specification will be called the "guidance equation."

Once it has been settled on what basis the burning of the rocket should be terminated and in what manner the angle of attack is to be controlled during the flight, the shape of the flight path is determined physically as well as mathematically. It is evident, therefore, that one should choose the cutoff criterion and the guidance equation in a way which will impress upon the resulting flight path certain desired characteristics.

In the case of surface-to-surface missiles, the desired characteristic is a high accuracy of hitting. It will be shown that this requirement determines essentially on what basis cutoff should be effected and how guidance might be carried out by radio stations after cutoff.

THE CALCULUS OF PERTURBATIONS

To apply the calculus of perturbations, one must assume that a solution of Eq. 1 has been found. This solution will be called the "standard trajectory." The calculus of perturbations, then, affords a relatively simple means of deriving from the standard trajectory a great number of others which differ from it somewhat in the following respect: the standard trajectory has been established on the basis of certain assumptions concerning, say, the weight, drag, lift of the missile, its burning time, the absence or presence of wind, etc. If any of these assumptions is varied slightly a new trajectory results which differs somewhat from the standard and can be derived from it by the calculus of perturbations.

This technique has been well utilized on conventional ballistics, but only to a limited degree in the ballistics of guided missiles, although the mathematical process is the same in both cases. One starts with the equations of motion in a form equivalent to Eqs. 1:

$$\begin{aligned} \dot{x} &= v_x, \\ \dot{y} &= v_y, \\ \dot{v}_x &= F(y, v_x, v_y, \alpha, t), \\ \dot{v}_y &= G(y, v_x, v_y, \alpha, t), \end{aligned} \tag{2}$$

where F and G are two functions describing the behavior of the projectile or missile under the combined effect of aerodynamic forces, thrust and gravity. A standard trajectory is then a solution

$$x = \bar{x}, \quad y = \bar{y}, \quad v_x = \bar{v}_x, \quad v_y = \bar{v}_y, \quad \alpha = \bar{\alpha} \tag{3}$$

of these equations, stating the path variables (and—for missiles—also the angle of attack) as functions of the time. Now assume that the functions F and G contain a parameter μ—the weight of the body, say—

and that the standard path (3) has been derived for $\mu = \bar{\mu}$. If the parameter now is changed somewhat to

$$\mu = \bar{\mu} + \delta\mu,$$

where $\delta\mu$ is a small quantity, then a corresponding small change will occur in the path variables:

$$\begin{aligned} x &= \bar{x} + \delta x, \quad y = \bar{y} + \delta y, \quad v_x = \bar{v}_x + \delta v_x, \\ v_y &= \bar{v}_y + \delta v_y, \quad \alpha = \bar{\alpha} + \delta\alpha. \end{aligned} \tag{4}$$

Moreover, the small perturbations will satisfy, to first order accuracy, equations

$$\begin{aligned} \frac{d}{dt}(\delta x) &= \delta v_x, \\ \frac{d}{dt}(\delta y) &= \delta v_y, \\ \frac{d}{dt}(\delta v_x) &= \frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial v_x}\delta v_x + \frac{\partial F}{\partial v_y}\delta v_y + \frac{\partial F}{\partial \alpha}\delta\alpha + \frac{\partial F}{\partial \mu}\delta\mu, \\ \frac{d}{dt}(\delta v_y) &= \frac{\partial G}{\partial y}\delta y + \frac{\partial G}{\partial v_x}\delta v_x + \frac{\partial G}{\partial v_y}\delta v_y + \frac{\partial G}{\partial \alpha}\delta\alpha + \frac{\partial G}{\partial \mu}\delta\mu, \end{aligned}$$

or briefly

$$\begin{aligned} \delta\dot{x} &= \delta v_x, \\ \delta\dot{y} &= \delta v_y, \\ \delta\dot{v}_x &= a_1\delta y + a_2\delta v_x + a_3\delta v_y + a_4\delta\alpha + a_5\delta\mu, \\ \delta\dot{v}_y &= b_1\delta y + b_2\delta v_x + b_3\delta v_y + b_4\delta\alpha + b_5\delta\mu. \end{aligned} \tag{5}$$

The partial derivatives in these equations are all evaluated for $x = \bar{x}$, $y = \bar{y}$, $\cdots$ $\mu = \bar{\mu}$, that is, using the variables of the standard trajectory. Explicit expressions for those derivatives are given at several places [2] for the case of conventional ballistics. If the projectile under consideration is a missile, one finds

$$\begin{aligned} a_1 &= \phi_T(v_x\cos\alpha + v_y\sin\alpha)\partial \ln F_T/\partial y && -v_x\Delta(P - QN) + v_y\Lambda(P - RN), \\ a_2 &= -\phi_T\frac{v_y}{v^2}(v_x\sin\alpha - v_y\cos\alpha) && -\frac{v_x^2}{v^2}\Delta(1+Q) + \frac{v_xv_y}{v^2}\Lambda(1+R) - \Delta, \\ a_3 &= \phi_T\frac{v_x}{v^2}(v_x\sin\alpha - v_y\cos\alpha) && -\frac{v_xv_y}{v^2}\Delta(1+Q) + \frac{v_y^2}{v^2}\Lambda(1+R) + \Lambda, \\ a_4 &= -\phi_T(v_x\sin\alpha - v_y\cos\alpha) && -v_x\Delta\partial \ln C_D/\partial\alpha + v_y\Lambda\partial \ln C_L/\partial\alpha, \\ b_1 &= -\phi_T(v_x\sin\alpha - v_y\cos\alpha)\partial \ln F_T/\partial y && -v_y\Delta(P - QN) - v_x\Lambda(P - RN), \\ b_2 &= -\phi_T\frac{v_y}{v^2}(v_x\cos\alpha + v_y\sin\alpha) && -\frac{v_xv_y}{v^2}\Delta(1+Q) - \frac{v_x^2}{v^2}\Lambda(1+R) - \Lambda, \\ b_3 &= \phi_T\frac{v_x}{v^2}(v_x\cos\alpha + v_y\sin\alpha) && -\frac{v_y^2}{v^2}\Delta(1+Q) - \frac{v_xv_y}{v^2}\Lambda(1+R) - \Delta, \\ b_4 &= -\phi_T(v_x\cos\alpha + v_y\sin\alpha) && -v_y\Delta\partial \ln C_D/\partial\alpha - v_x\Lambda\partial \ln C_L/\partial\alpha. \end{aligned} \tag{6}$$

[2] For example, F. R. MOULTON, "New Methods in Exterior Ballistics," Chicago, University of Chicago Press, 1926.

Here, N, P, Q, R, are used to denote the expressions:

$$N = \frac{1}{a}\frac{da}{dy}, \qquad P = \frac{1}{\rho}\frac{d\rho}{dy}, \qquad Q = \frac{M}{C_D}\frac{\partial C_D}{\partial M}, \qquad R = \frac{M}{C_L}\frac{\partial C_L}{\partial M}.$$

The partial derivatives a_5, b_5 depend, of course, on what parameter is being identified with μ. Several possibilities which are often encountered in design studies and similar investigations are:

Variations in the drag coefficient ($\mu = C_D$):

$$a_5 = -v_x\Delta/C_D, \qquad b_5 = -v_y\Delta/C_D.$$

Variations in the lift coefficient ($\mu = C_L$):

$$a_5 = v_y\Delta/C_L, \qquad b_5 = -v_x\Delta/C_L.$$

Variations in the structural weight ($\mu = W_s$):

$$a_5 = -\dot{v}_x/W, \qquad b_5 = -(\dot{v}_y + g)/W.$$

Variations in the specific impulse ($\mu = \tau$):

$$a_5 = \phi_T(v_x\cos\alpha + v_y\sin\alpha)/\tau, \qquad b_5 = -\phi_T(v_x\sin\alpha - v_y\cos\alpha)/\tau.$$

These coefficients as well as the ones of Eqs. 6 can be tabulated using the path variables of the standard trajectory. The numerical integration of the system (5) can then proceed without difficulty. The result will be a new trajectory showing the effect of the changed parameter.

For certain applications it is more convenient to solve the adjoint system rather than the original system of equations. This technique was suggested by Bliss [3] and is particularly useful in the problems which will be considered below. The system adjoint to (5) is

$$\begin{aligned} -\dot{\lambda}_1 &= 0 \\ -\dot{\lambda}_2 &= a_1\lambda_3 + b_1\lambda_4, \\ -\dot{\lambda}_3 &= \lambda_1 + a_2\lambda_3 + b_2\lambda_4, \\ -\dot{\lambda}_4 &= \lambda_2 + a_3\lambda_3 + b_3\lambda_4. \end{aligned} \tag{8}$$

It can be shown [3] that its solutions are ultimately connected with those of the original system by the relation

$$\begin{aligned} &[\lambda_1\delta x + \lambda_2\delta y + \lambda_3\delta v_x + \lambda_4\delta v_y]_{t=t_1} \\ &\quad = [\lambda_1\delta x + \lambda_2\delta y + \lambda_3\delta v_x + \lambda_4\delta v_y]_{t=t_2} \\ &\qquad + \int_{t_1}^{t_2}[(\lambda_3 a_4 + \lambda_4 b_4)\delta\alpha + (\lambda_3 a_5 + \lambda_4 b_5)\delta\mu]dt, \end{aligned} \tag{9}$$

where t_1 and t_2 are two arbitrary times of flight on the standard trajectory.

[3] G. A. BLISS, "Mathematics for Exterior Ballistics," New York, John Wiley & Sons, Inc., 1944.

THE CUTOFF CRITERION

In the case of the ground-to-ground missiles, the chief requirement is that of accuracy of hitting. That is, the flight should be so controlled that it should end at the target regardless of variations from one flight to the next. This implies, among other things, that the cutoff of the power flight should be effected at exactly the correct instant. A precise cutoff timing is, of course, of particular importance for missiles of the type of the German V-2, which are no longer controlled after cutoff. But it is also quite critical in the case of missiles in which control does not cease simultaneously with the burning. The correction of cutoff errors requires, in general, a good deal of maneuvering which must take place during the free flight, that is, in a virtual vacuum, where effective steering is either impossible or highly impractical.

The timing of the power cutoff would seem at first a rather straightforward matter. The range at which a target is located determines the standard trajectory along which the target is to be reached. Its usefulness is, however, somewhat hypothetical; for the probability that a missile under actual flight conditions would ever follow that standard trajectory is extremely small. Fluctuations in the performance of the rocket motor, for instance, will cause it to travel a path a good deal different from the expected one and, at the time of standard cutoff, the missile will in general be neither at the desired point $(\bar{x}_c, \bar{y}_c)$ nor travel with the desired velocity $(\bar{v}_{xc}, \bar{v}_{yc})$. Rather, its co-ordinates (x_c, y_c) and velocity components (v_{xc}, v_{yc}) will differ from the anticipated ones by δx_c, δy_c, δv_{xc}, δv_{yc}:

$$x_c = \bar{x}_c + \delta x_c, \quad y_c = \bar{y}_c + \delta y_c, \quad v_{xc} = \bar{v}_{xc} + \delta v_{xc}, \quad v_{yc} = \bar{v}_{yc} + \delta v_{yc}.$$

It is now desired to relocate the cutoff point in such a way that despite the departures δx_c, δy_c, δv_{xc}, δv_{yc}, the range of the missile remains unaffected. To insure this one can proceed as follows:

The range X is directly a function of the cutoff conditions,

$$X = X(x_c, y_c, v_{xc}, v_{yc}),$$

and the range error consequently can be written in the first order approximation as a linear combination of the departures, *viz*:

$$\begin{aligned} \delta X &= \frac{\partial X}{\partial x_c}\delta x_c + \frac{\partial X}{\partial y_c}\delta y_c + \frac{\partial X}{\partial v_{xc}}\delta v_{xc} + \frac{\partial X}{\partial v_{yc}}\delta v_{yc} \\ &= \lambda_1 \delta x_c + \lambda_2 \delta y_c + \lambda_3 \delta v_{xc} + \lambda_4 \delta v_{yc}. \end{aligned} \tag{10}$$

Moreover, as has been shown by Bliss,[3] the coefficients λ can be computed as solutions of the adjoint system (8). They are obtained by integrating that system "backward," that is, from the standard impact time T to the standard cutoff time t_c, using as "initial" conditions

$$\lambda_1 = 1, \quad \lambda_2 = \tan \omega, \quad \lambda_3 = \lambda_4 = 0 \quad \text{at } t = T. \tag{11}$$

This means that the coefficients λ are computable as soon as the standard trajectory has been established, and that one can (if that should be of interest) predict the impact error from the departures δx_c, δy_c, δv_{xc}, δv_{yc}.

In fact, it follows from an inspection of Eq. 10 that the coefficients λ may be interpreted as the range errors produced by unit departures of the cutoff conditions from standard.

The object of this derivation is, however, not to predict but to eliminate the impact error, that is, it is desired to insure

$$\delta X = 0.$$

The departure of the actual from the standard cutoff must, therefore, be so coordinated that

$$\lambda_1 \delta x_c + \lambda_2 \delta y_c + \lambda_3 \delta v_{xc} + \lambda_4 \delta v_{yc} = 0. \tag{12}$$

This can be accomplished in the following way: Assume that, by some—yet unspecified—equipment, the co-ordinates x, y and velocity components v_x, v_y of the missile were being recorded during power flight. Assume, furthermore, that these data were being superposed continuously to form the quantity

$$J = \lambda_1 x + \lambda_2 y + \lambda_3 v_x + \lambda_4 v_y, \tag{13}$$

and compared with a corresponding quantity

$$\bar{J} = \lambda_1 \bar{x}_c + \lambda_2 \bar{y}_c + \lambda_3 \bar{v}_{xc} + \lambda_4 \bar{v}_{yc}.$$

The latter is composed in the same manner of the standard cutoff conditions $\bar{x}_c$, $\bar{y}_c$, $\bar{v}_{xc}$, $\bar{v}_{yc}$ as J is composed of the actual missile co-ordinates x, y, v_x, v_y during flight. $\bar{J}$, however, involves only quantities which are directly evident from the standard trajectory and have been computed in advance.

Now, if the cutoff is effected when

$$J = \bar{J}, \tag{14}$$

then, according to Eq. 12 the range error will be zero. Hence, Eq. 14 constitutes the desired cutoff criterion which insures a hit regardless of variations in the power flight. A point at which Eq. 14 is fulfilled will exist along any trajectory since J is a monotonely increasing quantity during power flight and, therefore, must at some time reach the prescribed constant $\bar{J}$.

This line of reasoning, though in a somewhat different form, was actually used by the Germans in establishing the theoretical basis for the power cutoff of the V-2.[4] When it came to translating the method into practice, however, they found two other forms of the "cutoff constant" $\bar{J}$ more convenient.

[4] E. HECKMANN, "Ballistic Corrections of the Trajectories of Missile A4," Foreign Document Evaluation Section, Aberdeen Proving Grounds, Maryland, 1945.

The first form is best suited to the case in which the measurement of the path variables is done by a ground radio station. It is then more appropriate to use polar co-ordinates r, ϕ, $\dot{r}$, $\dot{\phi}$, with the pole at the radio station, rather than rectangular co-ordinates, and to convert the quantity J into the form

$$J = \mu_1 r + \mu_2 \phi + \mu_3 \dot{r} + \mu_4 \dot{\phi}.$$

The coefficients μ can be obtained from the λ by a simple co-ordinate transformation. Moreover, one is now free to dispose in the most convenient way of a newly introduced parameter: The location of the radio station. It was decided by the Germans to choose this location so that

$$\mu_4 = 0$$

because it was felt that ϕ was the most difficult quantity to measure accurately. It was found that, in this case, the radio station had to be placed at the location at which the tangent drawn to the path at the cutoff point pierced the ground plane.

A second alternative for measuring the path variables in J is presented by the use of integrating accelerometers. In the case of the V-2 missile it was decided that two such accelerometers should be used, one singly integrating and one doubly integrating. They were conceived as being mounted space-fixed (on a stabilized platform) at two angles γ and ϵ with the horizontal. These angles would be determined by the relations

$$\tan \gamma = \frac{\lambda_3}{\lambda_4}, \qquad \tan \epsilon = \frac{\lambda_1}{\lambda_2}.$$

In this arrangement, the doubly integrating accelerometer would measure the quantity

$$r_\epsilon = x \cos \epsilon + y \sin \epsilon = (\lambda_1 x + \lambda_2 y)/(\lambda_1^2 + \lambda_2^2)^{\frac{1}{2}},$$

and the singly integrating accelerometer the quantity

$$v_\gamma = v_x \cos \gamma + v_y \sin \gamma = (\lambda_3 v_x + \lambda_4 v_y)/(\lambda_3^2 + \lambda_4^2)^{\frac{1}{2}}.$$

Hence, if the outputs r_ϵ and v_γ of these two instruments were superposed in the ratio $(\lambda_1^2 + \lambda_2^2)^{\frac{1}{2}}/(\lambda_3^2 + \lambda_4^2)^{\frac{1}{2}}$, a quantity proportional to J would be obtained which can be used to release the cutoff signal.

It should be added at this point that neither of the two systems—radio or accelerometers—was ever employed in practice as established by theory. Rather, a number of simpler systems were derived from the theoretically accurate ones which were admittedly less precise but still insured a fair accuracy of hitting. These simplified systems were generally less expensive. They also were in some ways more logical. For it was found in the course of the test firings of the V-2 that even the most accurate system of generating the cutoff signal could not have

materially improved its accuracy of hitting. The chief errors actually arose after the cutoff signal had been released and the power plant of the missile supposedly had been shut down.

GUIDANCE EQUATIONS

If cutoff is effected according to the criterion (14) no range error will result, providing no further disturbances occur during the ensuing free flight. This provision is, of course, not generally fulfilled. In fact, the most serious difficulty arises frequently right after the release of the cutoff signal. Theoretically, the cutoff signal should entail the instantaneous shut-down of the power plant. Actually, this is not likely to happen. There usually develops some after-burning in the motor which continues to accelerate the missile beyond cutoff and may ultimately lead to a considerable range error.

In addition, the free flight is affected by winds, variations in air density, drag coefficient, missile oscillations, etc. As a result of all this, the impact will in general still be quite uncertain no matter how carefully the cutoff signal has been timed. If a high accuracy of hitting is desired, one is forced to resort to some guidance after cutoff.

The V-2 missile was conceived as a strategic weapon and was not intended for use against small targets. Hence, no guidance was provided after cutoff. The controls were returned to neutral and the flight continued like that of an ordinary projectile. This arrangement virtually eliminated the V-2 as a tactical weapon. The one time when it was so used—against the bridgehead of the first U. S. Army at Remagen—the mission probably was quite unsuccessful for those reasons.

It should be of some interest to inquire into how guidance could, at least theoretically, be effected after cutoff. Of the various schemes upon which one could embark in such speculation, remote control by radio is perhaps one of the more challenging. In principle, there are various ways of employing ground radio stations for the purpose of flight supervision, but all have certain common limitations. The most striking of these is the fact that it will, in general, be impossible or impractical to track the missile all the way to the impact. Radio waves which travel too close to the ground have highly uncertain propagation paths or may be intercepted altogether by the terrain. It will, therefore, be advisable not to plan on radio contact with the missile below a certain elevation angle. The altitude of the point at which remote guidance then ends will depend upon the location of the radio station (or stations). The closer they can be moved to the impact point the closer to the ground one will be able to track the missile. If one can assume a minimum elevation angle of about ten degrees and if the radio stations can be placed no nearer to the target than about 150 miles, guidance will be discontinued at an altitude of about twenty miles. At about fifty miles from the target, that altitude is reduced to seven miles, and

at ten miles distance it is only 1.4 miles. In any case, the guidance plan must be so laid that all predictable range and deflection errors have been eliminated by the time control is discontinued. The only impact deviations which cannot be prevented are then those which accumulate in the period between termination of control and impact.

The suppression of all predictable impact errors can be accomplished by a scheme which resembles in some ways the cutoff process. With every time t of flight, there are associated two quantities

$$J = \lambda_1 x + \lambda_2 y + \lambda_3 v_x + \lambda_4 v_y$$

and

$$J = \lambda_1 \bar{x} + \lambda_2 \bar{y} + \lambda_3 \bar{v}_x + \lambda_4 \bar{v}_y,$$

which could be formed and computed as during power flight—with one important difference: that the coefficients λ are now no longer constants. They are, however, again precomputed from the standard trajectory by the adjoint system (8) and are known functions of time. The same is true of $\bar{x}$, $\bar{y}$, $\bar{v}_x$, $\bar{v}_y$, which are the path variables of the standard trajectory. Hence, J is also known for any time of flight t. The quantities x, y, v_x, v_y, on the other hand, are the position co-ordinates and velocity components which are measured during flight. The difference between J and J

$$\begin{aligned}\delta J &= \lambda_1(x - \bar{x}) + \lambda_2(y - \bar{y}) + \lambda_3(v_x - \bar{v}_x) + \lambda_4(v_y - \bar{v}_y) \\ &= \lambda_1 \delta x + \lambda_2 \delta y + \lambda_3 \delta v_x + \lambda_4 \delta v_y,\end{aligned}$$

which can be computed at any time t, is—by definition of J—the range error which can be predicted at that time. This statement can be made somewhat more concise. According to Eq. 9 one must have

$$\begin{aligned}[\lambda_1 \delta x + \lambda_2 \delta y + \lambda_3 \delta v_x + \lambda_4 \delta v_y]_T \\ = [\lambda_1 \delta x + \lambda_2 \delta y + \lambda_3 \delta v_x + \lambda_4 \delta v_y]_t \\ + \int_t^T [(\lambda_3 a_4 + \lambda_4 b_4)\delta\alpha + (\lambda_3 a_5 + \lambda_4 b_5)\delta\mu]dt.\end{aligned}$$

Now, λ_1, λ_2, λ_3, λ_4, are the solutions of the adjoint system (8) if integrated with the initial conditions (11), that is,

$$\lambda_1 = 1, \quad \lambda_2 = \tan\omega, \quad \lambda_3 = \lambda_4 = 0, \quad \text{at } t = T.$$

Hence

$$\begin{aligned}[\delta x + \delta y \tan\omega]_T = \delta J|_t \\ + \int_t^T [(\lambda_3 a_4 + \lambda_4 b_4)\delta\alpha + (\lambda_3 a_5 + \lambda_4 b_5)\delta\mu]dt. \quad (16)\end{aligned}$$

The left-hand side of this relation can quickly be shown to be the first order range error δX (see Fig. 1): at the time T of standard impact, the missile flying a perturbed trajectory will have arrived at a point P with co-ordinates $(X + \delta x, \delta y)$. The impact point—to first order accuracy—

will then have the abscissa

$$X + \delta X = X + \delta x + \delta y \tan \omega.$$

Equation 16 can, therefore, be written

$$\delta X = \delta J|_t + \int_t^T [(\lambda_3 a_4 + \lambda_4 b_4)\delta\alpha + (\lambda_3 a_5 + \lambda_4 b_5)\delta\mu]dt. \quad (17)$$

Now let t be interpreted as a time shortly before the end of guidance. The integral on the right-hand side of (17) then sums all disturbances during the remainder of the flight from the time t till impact and constitutes a largely unpredictable quantity. The term $\delta J|_t$ on the other hand, includes the effect of all disturbances which have preceded t. It will, in general, be the dominant term of the right-hand side of (17), and this for two reasons.

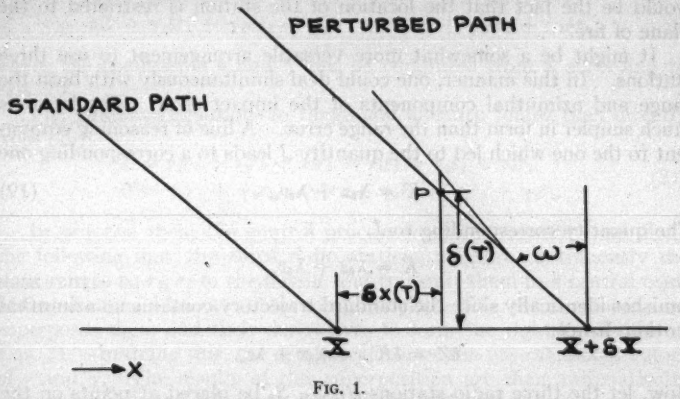

FIG. 1.

Firstly, the disturbances contained in the integral term are chiefly of aerodynamic nature (that is, wind, variations in missile drag and lift) and are relatively small. But—and this the second reason—even these small disturbances will be taken account of during the major portion of the flight until the end of control.

It is only during the unguided period of flight that they can accumulate an unpredictable range error. Accordingly, it is both necessary and reasonable to put

$$\delta X \doteq \delta J|_t.$$

Hence, to prevent the occurrence of a range error one must keep

$$\delta J|_t = 0.$$

The flight, in other words, is to be guided in such a way that, at all times

$$J = \lambda_1 x + \lambda_2 y + \lambda_3 v_x + \lambda_4 v_y. \quad (18)$$

This is the desired guidance equation. It requires the missile to adjust its angle of attack in such that Eq. 18 is continuously fulfilled.

In practice, the variables x, y, v_x, v_y cannot be measured conveniently by radio. One could perform the co-ordinate transformation which led from (13) to (15) and throw J into the form

$$J = \mu_1 r + \mu_2 \phi + \mu_3 \dot{r} + \lambda_4 \dot{\phi},$$

where r and $\dot{r}$ would stand for the slant range and its rate, and ϕ and $\dot{\phi}$ for the elevation angle and its rate. It would be the form appropriate for a radio station placed exactly in the plane of fire. This sort of arrangement would have certain disadvantages, however. It could, first of all, eliminate only the range error. An independent system would have to remove the error in deflection. A second drawback would be the fact that the location of the station is restricted to the plane of fire.

It might be a somewhat more versatile arrangement to use three stations. In this manner, one could deal simultaneously with both the range and azimuthal components of the impact error. The latter is much simpler in form than the range error. A line of reasoning equivalent to the one which led to the quantity J leads to a corresponding one

$$K = \lambda_5 z + \lambda_6 v_z. \tag{19}$$

The quantity corresponding to $\dot{J}$,

$$\dot{K} = \lambda_5 \dot{z} + \lambda_6 \dot{v}_z$$

vanishes identically since the standard trajectory contains no azimuthal motion; hence,

$$\delta Z \doteq \delta K|_t = \lambda_5 z + \lambda v_z.$$

Now, let the three radio stations S_1, S_2, S_3 be placed at points on the ground with the co-ordinates

$$S_1(x_1, 0, z_1), \quad S_2(x_2, 0, z_2), \quad S_3(x_3, 0, z_3),$$

and let r_1, r_2, r_3 be the slant ranges from the stations to the missile. Then

$$r_i^2 = (x - x_i)^2 + y^2 + (z - z_i)^2, \quad (i = 1, 2, 3).$$

The impact errors δX and δZ in terms of the tri-polar co-ordinates r_1, r_2, r_3, are then

$$\delta X = \sum_{i=1}^{3} \frac{\partial X}{\partial r_i} \delta r_i + \sum_{i=1}^{3} \frac{\partial X}{\partial \dot{r}_i} \delta \dot{r}_i,$$

$$\delta Z = \sum_{i=1}^{3} \frac{\partial Z}{\partial r_i} \delta r_i + \sum_{i=1}^{3} \frac{\partial Z}{\partial \dot{r}_i} \partial \dot{r}_i,$$

where

$$\begin{aligned}
\nu_i &= \frac{\partial X}{\partial r_i} = \frac{\partial X}{\partial x}\frac{\partial x}{\partial r_i} + \frac{\partial X}{\partial y}\frac{\partial y}{\partial r_i} = \lambda_1 \frac{\bar{r}_i}{\bar{x} - x_i} + \lambda_2 \frac{\bar{r}_i}{\bar{y}}, \quad (i = 1, 2, 3)\\
\nu_i &= \frac{\partial X}{\partial \dot{r}_i} = \frac{\partial X}{\partial v_x}\frac{\partial v_x}{\partial \dot{r}_i} + \frac{\partial X}{\partial v_y}\frac{\partial v_y}{\partial \dot{r}_i} = \lambda_3 \frac{\bar{r}_i}{\bar{x} - x_i} + \lambda_4 \frac{\bar{r}_i}{\bar{y}}, \quad (i = 4, 5, 6)\\
\gamma_i &= \frac{\partial Z}{\partial r_i} = \frac{\partial Z}{\partial z}\frac{\partial z}{\partial r_i} = \lambda_5 \frac{\bar{r}_i}{z_i}, \quad (i = 1, 2, 3)\\
\gamma_i &= \frac{\partial Z}{\partial \dot{r}_i} = \frac{\partial Z}{\partial v_z}\frac{\partial v_z}{\partial \dot{r}_i} = \lambda_6 \frac{\bar{r}_i}{z_i}. \quad (i = 4, 5, 6)
\end{aligned} \tag{20}$$

Range and deflection errors can, therefore, be written in the form

$$\begin{aligned}
\delta X &= \nu_1\delta r_1 + \nu_2\delta r_2 + \nu_3\delta r_3 + \nu_4\delta \dot{r}_1 + \nu_5\delta \dot{r}_2 + \nu_6\delta \dot{r}_3,\\
\delta Z &= \gamma_1\delta r_1 + \gamma_2\delta r_2 + \gamma_3\delta r_3 + \gamma_4\delta \dot{r}_1 + \gamma_5\delta \dot{r}_2 + \gamma_6\delta \dot{r}_3,
\end{aligned}$$

where the coefficients ν and γ can be computed from λ according to Eqs. 20.

The guidance equations in range and azimuth follow then directly from (20)

$$\begin{aligned}
J &= \nu_1 r_1 + \nu_2 r_2 + \nu_3 r_3 + \nu_4 \dot{r}_1 + \nu_5 \dot{r}_2 + \nu_6 \dot{r}_3,\\
0 &= \gamma_1 r_1 + \gamma_2 r_2 + \gamma_3 r_3 + \gamma_4 \dot{r}_1 + \gamma_5 \dot{r}_2 + \gamma_6 \dot{r}_3.
\end{aligned} \tag{21}$$

In practice then, the control process would conceivably run along the following line: the three radio stations measure continuously the slant ranges r_1, r_2, r_3 to the missile and transmit them to a central computing station. An automatic computer there differentiates them, and superposes them and their derivatives to form the right-hand sides of Eqs. 21. In doing this, the computer utilizes the pre-calculated values of ν and γ. The results of this superposition are then automatically compared with the left-hand sides of Eqs. 21 and any discrepancy is transmitted to the missile as an error signal. The missile now adjusts its angle of attack in pitch and yaw to reduce the errors to zero. This process should be completed at some time before control ends. The missile will then have been guided into a stable path which will terminate very close to the target.

LIST OF SYMBOLS

a = velocity of sound.
a_i = coefficients, defined in Eqs. 5.
b_i = coefficients, defined in Eqs. 5.
C_D = drag coefficient.
C_L = lift coefficient.
D = drag force.
F_T = thrust force.
g = acceleration of gravity.
J = guidance function in range (Eq. 10).

K = guidance function in deflection (Eq. 19).
L = lift force.
M = Mach number.
N = quantity introduced in Eq. 6.
P = quantity introduced in Eq. 6.
Q = quantity introduced in Eq. 6.
R = quantity introduced in Eq. 6.
r, r_i = slant ranges.
t = time of flight.
T = time of impact.
v_x, v_y, v_z = missile velocity components.
W = total instantaneous missile weight.
W_s = structural weight.
x, y, z = missile position co-ordinates (range, altitude, deflection).
X = range of fire.
α = angle of attack.
γ = angle of single integration of acceleration.
γ_i = coefficients defined by Eq. 20; (i = 1–6).
δ = denotes small variation of a quantity.
Δ = quantity introduced in Eq. 1.
ϵ = angle of double integration of acceleration.
λ_i = coefficients defined by Eq. 8 (i = 1, 2, 3, 4).
μ_i = coefficients derived from λ_i (i = 1, 2, 3, 4).
ν_i = coefficients defined by Eq. 20 (i = 1–6).
ρ = air density.
τ = specific impulse.
ϕ = angle of elevation.
ω = angle of impact.
Subscript c refers quantity to cutoff.
Bar above a quantity refers it to a standard trajectory.

THERMODYNAMIC PROPERTIES OF REACTIVE GAS MIXTURES

BY

ASHLEY S. CAMPBELL [1]

By the term "reactive gas mixture" will be understood a mixture of gases in which one or more chemical reactions take place simultaneously, the composition of the mixture being at all times in a state of chemical equilibrium. Opposed to reactive gas mixtures we have inert gas mixtures, in which no chemical reaction is possible, so that the question of chemical equilibrium does not arise. Between these two cases we may consider a third, namely, a mixture of gases capable of chemical activity with one another, but which for some reason do not react. For example, oxygen and hydrogen do not react at room temperature, and under this condition represent a mixture of the third type. Upon heating to an elevated temperature they react vigorously to produce water vapor, and establish chemical equilibrium. If this equilibrium mixture is cooled, chemical equilibrium will be maintained for a certain time but eventually a temperature is reached below which there is no further chemical activity. We have then a gas mixture which is reactive at high temperature and effectively inert at low temperature.

Now the thermodynamic properties of inert mixtures are simply stated in terms of the properties of constituent gases. The properties of active gas mixtures have not been extensively studied. Zemansky [2] has shown a formulation for the heat capacity. The purpose of this study is to begin an investigation of other properties, particular emphasis being placed on a comparison with a similar inert system. The reason for this emphasis arises from the obvious fact that inert gas mixtures are easy to handle numerically, while reactive gas mixtures always involve a composition change, which is seldom a simple matter to compute.

In particular we investigate the following situation in a simple mixture. We have on the one hand a mixture of reactive gases in equilibrium and we disturb the system slightly, and compute the changes attending the disturbance. Now on the other hand consider a system composed of the same composition, at the same initial condition. If for this second system we stipulate that the composition remain fixed, so the system becomes an inert mixture, the changes attending the same type of disturbance will in general be quite different. To be precise,

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[2] MARK WALDO ZEMANSKY, "Heat and Thermodynamics," second edition, New York, McGraw-Hill Book Co., 1943, p. 350.

we isentropically increase the temperature of both systems by the same amount and ask which system undergoes the larger change in pressure. A similar question has been discussed by Altman and Penner [3] who considered a rocket propelled by hydrogen gas, and inquired into the comparative behavior of the gas depending upon whether or not chemical reaction occurred during the expansion.

By limiting the present discussion so that only one gas reaction takes place, and further that only those gases taking place in the reaction are present, it has been found that gas reactions may be conveniently divided into three groups, each group having an essentially different behavior when compared with its inert counter-system. This is clearly a very sharp limitation for few systems of practical importance contain only a single reaction. The precise manner in which the composition enters the problem is clearly shown by the appearance of a composition function which is closely related to the mobility of the system.

ENTROPY AND THE HEAT CAPACITIES

It is convenient to begin by formulating expressions for the entropy. The analysis can be made sufficiently general by considering a four-gas reaction. Denote the gases by A_1, A_2, A_3, and A_4, and let the reaction be

$$\nu_1 A_1 + \nu_2 A_2 \rightleftarrows \nu_3 A_3 + \nu_4 A_4,$$

where the ν's are stoichiometric coefficients. According to Gibbs' theorem the total entropy, S, of an arbitrary mixture of ideal gases is

$$S = \sum_i n_i s_i, \tag{1}$$

where the summation extends over all gases. Here n denotes mole number and s denotes partial molal entropy. The change in total entropy is

$$dS = \sum_i n_i ds_i + \sum_i s_i dn_i. \tag{2}$$

Equation 2 is the point of departure between inert and reactive gas mixtures, for in the former case the second term of (2) is zero.

Since we imagine the gas mixture isolated from its surroundings the mole numbers may not vary independently, but rather

$$dn_1 : dn_2 : dn_3 : dn_4 = -\nu_1 : -\nu_2 : \nu_3 : \nu_4. \tag{3}$$

Clearly the relation between any pair of mole numbers is linear, so that (3) may be integrated and we have

$$\begin{aligned} n_1 &= \nu_1(1 - \epsilon) + a_1, \\ n_2 &= \nu_2(1 - \epsilon) + a_2, \\ n_3 &= \nu_3\epsilon + a_3, \\ n_4 &= \nu_4\epsilon + a_4. \end{aligned} \tag{4}$$

[3] S. S. PENNER AND D. ALTMAN, "Adiabatic Flow of Hydrogen Gas Through a Rocket Nozzle with and without Composition Changes," JOUR. FRANKLIN INST., Vol. 245, p. 421 (1948).

The mole numbers are thus stated in terms of a single variable, ϵ, the so-called "extent of reaction." Obviously ϵ is limited to positive values between zero and unity. The integrations constants, $a_1, \cdots, a_4$, are constant for any one mixture; their values fix the molecular composition. All four constants are included for the sake of generality, but it is clear that the constants for at least two of the gases must be zero, for if the reaction proceeds to the limit in either direction at least one gas must be consumed entirely. The total mole number, N, is now

$$N = \sum_i n_i = \nu_1 + \nu_2 + \epsilon\Delta\nu + \sum_i a_i \tag{5}$$

and

$$dN = \Delta\nu d\epsilon, \tag{6}$$

where

$$\Delta\nu = (\nu_3 + \nu_4) - (\nu_1 + \nu_2).$$

Introducing the partial molal entropy in terms of temperature and partial pressure,

$$ds_i = C_{P_i}\frac{dT}{T} - R\frac{dp_i}{p_i} \tag{7}$$

we may express the total entropy in terms of total pressure and temperature. Substitute (7) in (2) and one obtains after simplification

$$\frac{dS}{N} = C_P\frac{dT}{T} - R\frac{dP}{P} + \Delta s\frac{d\epsilon}{N}, \tag{8}$$

where

$$C_P = \sum_i x_i C_{P_i}, \qquad x_i = \frac{n_i}{N},$$

$$P = \sum_i p_i,$$

$$\Delta s = (\nu_3 s_3 + \nu_4 s_4) - (\nu_1 s_1 + \nu_2 s_2),$$

and R and T are the universal gas constant and absolute temperature, respectively. Now, since chemical equilibrium prevails at all times the composition is uniquely determined by total pressure and temperature, according to the Mass Action Law. Thus

$$\frac{n_3^{\nu_3}\cdot n_4^{\nu_4}}{n_1^{\nu_1}\cdot n_2^{\nu_2}}\left(\frac{P}{N}\right)^{\Delta\nu} = K, \tag{9}$$

where the reaction constant K is a function of temperature only. Differentiating (9) logarithmically

$$\nu_3\frac{dn_3}{n_3} + \nu_4\frac{dn_4}{n_4} - \nu_1\frac{dn_1}{n_1} - \nu_2\frac{dn_2}{n_2} = \frac{dK}{K} - \Delta\nu\frac{dP}{P} + \Delta\nu\frac{dN}{N}.$$

The left-hand side may be simplified by the use of Eqs. 4, while (6) may be substituted in the last term on the right-hand side. Collecting terms,

and introducing mole fractions, we have

$$\left[\frac{\nu_1^2}{x_1}+\frac{\nu_2^2}{x_2}+\frac{\nu_3^2}{x_3}+\frac{\nu_4^2}{x_4}-(\Delta\nu)^2\right]\frac{d\epsilon}{N}=\frac{dK}{K}-\Delta\nu\frac{dP}{P}. \tag{10}$$

To eliminate the reaction constant introduce van't Hoff's equation

$$\frac{dK}{K}=\frac{\Delta H}{RT}\cdot\frac{dT}{T},$$

where

$$\Delta H=(\nu_3\bar{h}_3+\nu_4\bar{h}_4)-(\nu_1\bar{h}_1+\nu_2\bar{h}_2),$$

that is, ΔH is the constant-pressure heat of reaction appropriate to temperature T. Adopting the notation

$$\frac{1}{\Psi}=\left[\frac{\nu_1^2}{x_1}+\frac{\nu_2^2}{x_2}+\frac{\nu_3^2}{x_3}+\frac{\nu_4^2}{x_4}-(\Delta\nu)^2\right] \tag{11}$$

(10) takes the form

$$\frac{d\epsilon}{N}=\Psi\left[\frac{\Delta H}{RT}\cdot\frac{dT}{T}-\Delta\nu\frac{dP}{P}\right]. \tag{12}$$

As a consequence of chemical equilibrium the entropy change, Δs, occurring in (8) is related to the heat of reaction, ΔH,

$$\Delta s=\frac{\Delta H}{T}. \tag{13}$$

Now eliminate $(d\epsilon)$ in (8) by substituting (12), and make use of (13). The resulting expression is

$$\frac{dS}{N}=\left[C_P+\frac{\Psi(\Delta H)^2}{RT^2}\right]\frac{dT}{T}-R\left[1+\frac{\Psi\Delta H\Delta\nu}{RT}\right]\frac{dP}{P}. \tag{14}$$

The analogous expression with temperature and volume, V, as independent variables is obtained by introducing the equation of state

$$\frac{dP}{P}+\frac{dV}{V}=\Delta\nu\frac{d\epsilon}{N}+\frac{dT}{T} \tag{15}$$

into (12) to eliminate dP. This leads to

$$\frac{d\epsilon}{N}=\Psi\left[\left(\frac{\Delta H}{RT}-\Delta\nu\right)\frac{dT}{T}+\Delta\nu\frac{dV}{V}-(\Delta\nu)^2\frac{d\epsilon}{N}\right].$$

But

$$\begin{aligned}\frac{\Delta H}{RT}-\Delta\nu&=\frac{\Delta H-RT\Delta\nu}{RT}\\&=\frac{\nu_3(\bar{h}_3-RT)+\nu_4(\bar{h}_4-RT)-\nu_1(\bar{h}_1-RT)-\nu_2(\bar{h}_2-RT)}{RT}\\&=\frac{\Delta U}{RT},\end{aligned}$$

where ΔU is the constant-volume heat of reaction appropriate to temperature T. Collecting terms we then have

$$\frac{d\epsilon}{N} = \Psi'\left[\frac{\Delta U}{RT}\frac{dT}{T} + \Delta\nu\frac{dV}{V}\right], \tag{16}$$

where now

$$\frac{1}{\Psi'} = \frac{1}{\Psi} + (\Delta\nu)^2. \tag{17}$$

Equations 15 and 16 may now be substituted into (8) to eliminate dP and $d\epsilon$, with the result,

$$\frac{dS}{N} = \left[C_V + \frac{\Psi'(\Delta U)^2}{RT^2}\right]\frac{dT}{T} + R\left[1 + \frac{\Psi'\Delta U\Delta\nu}{RT}\right]\frac{dV}{V}, \tag{18}$$

in which we have the usual relation,

$$C_V = \sum_i x_i C_{V_i} = C_P - R.$$

The constant-pressure and constant-volume heat capacities are obtained directly from (14) and (18) in accordance with their definition.

$$\begin{aligned} C_P^* &= \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_P = C_P + \frac{\Psi(\Delta H)^2}{RT^2}, \\ C_V^* &= \frac{T}{N}\left(\frac{\partial S}{\partial T}\right)_V = C_V + \frac{\Psi'(\Delta U)^2}{RT^2}. \end{aligned} \tag{19}$$

That these expressions are physically correct may be shown by introducing for Ψ and Ψ'

$$\Psi = \frac{RT^2}{N\Delta H}\cdot\left(\frac{\partial\epsilon}{\partial T}\right)_P \quad \text{from (12)}$$

and

$$\Psi' = \frac{RT^2}{N\Delta U}\left(\frac{\partial\epsilon}{\partial T}\right)_V \quad \text{from (16).}$$

Then (19) take the form

$$\begin{aligned} C_P^* &= C_P + \frac{\Delta H}{N}\left(\frac{\partial\epsilon}{\partial T}\right)_P, \\ C_V^* &= C_V + \frac{\Delta U}{N}\left(\frac{\partial\epsilon}{\partial T}\right)_V. \end{aligned} \tag{20}$$

In (20) the products $\Delta H d\epsilon$ and $\Delta U d\epsilon$ represent the heat which must be supplied to accomplish chemical reaction of amount $d\epsilon$.

Forming the difference between heat capacities,

$$\begin{aligned} C_P^* - C_V^* &= C_P - C_V + \frac{\Psi(\Delta H)^2}{RT^2} - \frac{\Psi'(\Delta U)^2}{RT^2} \\ &= R\left[1 + \Psi\left(\frac{\Delta H}{RT}\right)^2 - \Psi'\left(\frac{\Delta U}{RT}\right)^2\right]. \end{aligned}$$

Substituting for Ψ' and $\frac{\Delta U}{RT}$,

$$C_P^* - C_V^* = R \frac{\left(1 + \frac{\Psi \Delta H \Delta \nu}{RT}\right)^2}{1 + \Psi(\Delta \nu)^2}. \tag{21}$$

Equation 21 may be obtained directly from the thermodynamic relation,

$$C_P^* - C_V^* = \frac{T}{N}\left(\frac{\partial V}{\partial T}\right)_P \cdot \left(\frac{\partial P}{\partial T}\right)_V.$$

The compressibility and thermal expansion coefficient are obtained by straightforward manipulation,

$$\begin{aligned} -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T &= \frac{1 + \Psi(\Delta \nu)^2}{P}, \\ \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P &= \frac{1 + \Psi \Delta \nu \frac{\Delta H}{RT}}{T}. \end{aligned} \tag{22}$$

The differential expressions for internal energy and enthalpy are obtained by eliminating the entropy, by (14) and (18), from the equations

$$\begin{aligned} dU &= TdS - PdV \\ dH &= TdS + VdP, \end{aligned}$$

with the results,

$$\begin{aligned} \frac{dU}{N} &= C_V^* dT + \Psi' \Delta U \Delta \nu \frac{dV}{V}, \\ \frac{dH}{N} &= C_P^* dT - \Psi \Delta H \Delta \nu \frac{dP}{P}. \end{aligned} \tag{23}$$

The equations for entropy, heat capacities, compressibility, thermal expansion, internal energy and enthalpy, as developed for gaseous systems including chemical reaction, contain terms which do not arise when the gases are inert. It is interesting to note that even though the individual gases are ideal, the internal energy and enthalpy of the mixture are not functions of temperature alone. For certain types of gaseous reactions the equations simplify. If for example there is no mole change due to chemical reaction, that is $\Delta \nu = 0$, the compressibility and thermal expansion reduce to the inert gas mixture form, while internal energy and enthalpy are functions of temperature only. The latter simplification is due to the fact that for $\Delta \nu = 0$, composition is a function of temperature alone. If $\Delta U = \Delta H = 0$, so that no thermal effects are associated with the reaction, then all equations, with the exception of compressibility, reduce to the inert gas form. Chemical

change without thermal effect is a phenomenon which does not appear to exist, however.

The equations contain Ψ and Ψ', and since these are new concepts we shall try to show the physical significance of these quantities.

INFORMATION ON Ψ

Equations 12 and 16 predict the manner in which the extent of reaction changes as a result of independent changes in pressure and temperature, or volume and temperature. They are therefore analytical formulations of Le Chatelier's Principle. Recognizing this one may show immediately that Ψ is always positive (Ψ' is obviously positive). However, Le Chatelier's Principle is derived from the condition that Gibbs function (free energy), G, is a minimum, when a system constrained to constant pressure and temperature reaches equilibrium. The Gibbs function is defined as

$$G = H - TS = \sum_i n_i \mu_i,$$

where μ_i are the chemical potentials, which have the following form for ideal gases,

$$\mu_i = \mu_i^\circ(P, T) + RT \ln x_i. \tag{24}$$

Differentiating G with respect to ϵ, at constant P and T, we have

$$\left(\frac{\partial G}{\partial \epsilon}\right)_{P,T} = \sum_i n_i \left(\frac{\partial \mu_i}{\partial \epsilon}\right)_{P,T} + \sum_i \mu_i \frac{dn_i}{d\epsilon}.$$

Using the definition of μ_i (24), the first term is zero, for

$$\begin{aligned} \sum_i n_i \left(\frac{\partial \mu_i}{\partial \epsilon}\right)_{P,T} &= RT \sum_i \frac{n_i}{x_i} \cdot \frac{dx_i}{d\epsilon}, \\ &= NRT \sum_i \frac{dx_i}{d\epsilon}, \\ &= NRT \frac{d}{d\epsilon} \left(\sum_i x_i\right) = 0. \end{aligned}$$

Consequently

$$\begin{aligned} \left(\frac{\partial G}{\partial \epsilon}\right)_{P,T} &= (\nu_3\mu_3 + \nu_4\mu_4) - (\nu_1\mu_1 + \nu_2\mu_2), \\ &= 0 \quad \text{(at equilibrium)}. \end{aligned} \tag{25}$$

Differentiating again

$$\left(\frac{\partial^2 G}{\partial \epsilon^2}\right)_{P,T} = \sum_i \left[\left(\frac{\partial \mu_i}{\partial \epsilon}\right)_{P,T} \cdot \frac{dn_i}{d\epsilon} + \mu_i \frac{d^2 n_i}{d\epsilon^2} \right].$$

The second term is zero since the mole numbers are linear in ϵ. Thus

$$\left(\frac{\partial^2 G}{\partial \epsilon^2}\right)_{P,T} = RT \sum_i \frac{1}{x_i}\frac{dx_i}{d\epsilon}\cdot\frac{dn_i}{d\epsilon},$$

$$= RT \sum_i \left[\frac{1}{n_i}\left(\frac{dn_i}{d\epsilon}\right)^2 - \frac{1}{N}\frac{dN}{d\epsilon}\cdot\frac{dn_i}{d\epsilon}\right],$$

$$= \frac{RT}{N}\left[\sum_i \frac{\nu_i^2}{x_i} - (\Delta\nu)^2\right],$$

$$= \frac{RT}{N\Psi} \quad \text{(at equilibrium)}. \tag{26}$$

Since G is a minimum the second derivative is positive, and thus Ψ is positive for all reactions.

If we were to maintain a gas mixture at constant P and T and then vary the composition, ϵ and G would both vary. According to (26) Ψ is proportional to the radius of curvature of the G *vs.* ϵ curve at the point G = minimum. For those points on the curve other than G = minimum the slope of the curve (25), will not be zero. The slope of the G *vs.* ϵ curve represents the "net chemical force" driving the system toward equilibrium, and the bracketed expressions of (25) represent the "driving" forces of the product and reactant gases. Denoting by Y the net driving force

$$Y = (\nu_3\mu_3 + \nu_4\mu_4) - (\nu_1\mu_1 + \nu_2\mu_2)$$

we may write for (26),

$$\Psi = \frac{RT}{N}\left(\frac{\partial \epsilon}{\partial Y}\right)_{P,T}, \qquad Y = 0.$$

Since the derivative is evaluated for $Y = 0$, we could introduce y, the net force per mole of mixture

$$y = \frac{Y}{N}$$

and have

$$\Psi = \frac{RT}{N^2}\left(\frac{\partial \epsilon}{\partial y}\right)_{P,T}, \qquad y = 0.$$

Inspection of the definition of Ψ in (11) shows that for a given composition the numerical value depends upon the particular choice of stoichiometric coefficients. If we arbitrarily select the quantity $(\sum_i \nu_i)^2$ as a multiplying factor, and write

$$\frac{1}{\Psi} = (\sum_i \nu_i)^2 \frac{1}{\psi},$$

where now

$$\frac{1}{\psi} = \frac{1}{(\sum_i \nu_i)^2}\left[\sum_i \frac{\nu_i^2}{x_i} - (\Delta\nu)^2\right],$$

then the numerical value of ψ depends only on the composition, and is independent of the particular choice of stoichiometric coefficients. Thus

$$\psi = RT\left(\frac{\sum_i \nu_i}{N}\right)^2\left(\frac{\partial\epsilon}{\partial y}\right)_{P,T}, \qquad y = 0. \tag{27}$$

This equation contains only intensive quantities. It shows that ψ is a state function, associated with the equilibrium state of the system. It is directly proportional to the amount of chemical reaction ($d\epsilon$), and inversely proportional to the force (dy) which must be exerted on the system to produce this amount of reaction. Thus ψ is a measure of "mobility" of the system.

In a similar way, starting with the Helmholtz function, $F = U - TS$, one may show that Ψ' is related to the mobility, with constant volume and temperature constraints on the system.

In the expressions for the heat capacities, for example (19), Ψ and Ψ' occur as multipliers in additive terms. Thus a chemical system, which because of its momentary composition is very mobile (large Ψ and Ψ'), will have heat capacities appreciably larger than the corresponding heat capacities of an inert system with similar composition. It is interesting to note that chemical systems always have larger heat capacity.

If the integration constants a_i in (4) are set equal to zero, Ψ has a simple form in terms of ϵ,

$$\Psi = \frac{\epsilon(1-\epsilon)}{(\nu_1+\nu_2)(\nu_3+\nu_4)},$$

which has its maximum value

$$\Psi_{max} = \frac{1}{4(\nu_1+\nu_2)(\nu_3+\nu_4)}, \qquad \text{at } \epsilon = \tfrac{1}{2}.$$

Thus if no gas is present in "excess" ($a_i = 0$), the mobility is a maximum when the reaction is at the halfway point between products and reactants. The quantity ψ introduced above has a maximum value

$$\psi_{max} = \frac{1}{4}\left[\frac{\nu_1+\nu_2}{\nu_3+\nu_4} + \frac{\nu_3+\nu_4}{\nu_1+\nu_2} + 2\right],$$

which is independent of the choice of stoichiometric coefficients.

ISENTROPIC CHANGES OF STATE

Since isentropic changes of state are of particular interest we shall study them in some detail. We introduce the notations

$$\left.\begin{aligned}\lambda &= \frac{1+\frac{\Psi(\Delta H)^2}{C_P RT^2}}{1+\frac{\Psi \Delta H \Delta\nu}{RT}} \\ \lambda' &= \frac{1+\frac{\Psi'(\Delta U)^2}{C_V RT^2}}{1+\frac{\Psi' \Delta U \Delta\nu}{RT}} \\ &= [1+\Psi(\Delta\nu)^2]\frac{1+\frac{\Psi'(\Delta U)^2}{C_V RT^2}}{1+\frac{\Psi \Delta H \Delta\nu}{RT}},\end{aligned}\right\} \tag{28}$$

where the second form for λ' is obtained by expressing Ψ' and ΔU in terms of Ψ and ΔH. From the equations for entropy (14) and (18), obtain the following differential quotients,

$$\left.\begin{aligned}\left(\frac{\partial P}{\partial T}\right)_s &= \frac{P}{T}\cdot\frac{C_P}{R}\cdot\lambda \\ \left(\frac{\partial V}{\partial T}\right)_s &= -\frac{V}{T}\frac{C_V}{R}\lambda' \\ \left(\frac{\partial P}{\partial V}\right)_s &= -\frac{P}{V}\cdot\gamma\cdot\frac{\lambda}{\lambda'}, \qquad \gamma = \frac{C_P}{C_V}.\end{aligned}\right\} \tag{29}$$

From the equations for internal energy (23) write

$$\frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_s = C_V^* + \frac{\Psi' \Delta U \Delta\nu}{V}\left(\frac{\partial V}{\partial T}\right)_s.$$

Eliminating the differential coefficient on the right, this may be reduced to

$$\frac{1}{N}\left(\frac{\partial U}{\partial T}\right)_s = \lambda' C_V.$$

Similarly,

$$\frac{1}{N}\left(\frac{\partial H}{\partial T}\right)_s = \lambda C_P. \tag{30}$$

One notes that for $\lambda = \lambda' = 1$, both (29) and (30) reduce to the inert gas form. Consequently the comparative behavior of two gas mixtures with identical composition, one considered chemically active, while the other is inert, may be studied by observing the values of λ and λ'.

From inspection of (28) we must consider three cases:

$$\begin{aligned} &\text{(I)} \quad \Delta H \Delta\nu = 0, \\ &\text{(II)} \quad \Delta H \Delta\nu > 0, \\ &\text{(III)} \quad \Delta H \Delta\nu < 0. \end{aligned}$$

Thus all reversible gas reactions are divided into three categories. Any given reaction belongs to only one category. We exclude the possibility of having ΔH change sign. Suppose $\Delta H > 0$ for low temperature and $\Delta H < 0$ for high temperature. Then according to van't Hoff's equation, following (10), starting at low temperature K would increase as the temperature increased, until one reached the temperature at which $\Delta H = 0$. Thereafter, for increasing temperature K would decrease. This means that if one were to heat a mixture of gases at constant pressure, starting at low temperature where the composition is composed largely of reactant gases, the concentration of product gases gradually increases with increasing T up to a certain point ($\Delta H = 0$), after which further increases in T result in a decrease of product gas concentrations. Thus one ends up at a high temperature with the composition appropriate to a low temperature. This contradicts experience.

Case I: $\Delta H \Delta\nu = 0$, *or simply* $\Delta\nu = 0$

The reaction does not change the total mole number. Thus $\Delta H = \Delta U$, and $\Psi = \Psi'$. And since $C_P > C_V$

$$\lambda' > \lambda > 1.$$

Case II: $\Delta H \Delta\nu > 0$

Thus $\Delta\nu$ and ΔH have the same sign. Both λ and λ' are positive. They may be equal however. This is found to occur when

$$\frac{\Delta H}{RT\Delta\nu} = \frac{\gamma}{\gamma - 1} \tag{31}$$

and for this condition $\lambda = \lambda' = 1$.

The left-hand side of (31) is a function of temperature only, while the right-hand side is a function of pressure and temperature. Consequently (31) represents a line in the (P, T) plane. This line divides the plane into two regions: in one region

$$\frac{\Delta H}{RT\Delta\nu} > \frac{\gamma}{\gamma - 1}, \quad \text{and} \quad \lambda, \lambda' > 1, \tag{32}$$

while in the other

$$\frac{\Delta H}{RT\Delta\nu} < \frac{\gamma}{\gamma - 1}, \quad \text{and} \quad \lambda, \lambda' < 1.$$

Since ΔH has a relatively small temperature coefficient, the first condition of (32) applies to the low temperature side of the line (31).

From (12) we have

$$\left(\frac{\partial P}{\partial T}\right)_s = \frac{P}{T}\cdot\frac{\Delta H}{RT\Delta\nu},$$

while from the first of (29), for $\lambda = 1$,

$$\left(\frac{\partial P}{\partial T}\right)_s = \frac{P}{T}\cdot\frac{\gamma}{\gamma - 1}.$$

Substituting the right-hand sides of these expressions into (31)

$$\left(\frac{\partial P}{\partial T}\right)_s = \left(\frac{\partial P}{\partial T}\right)_s, \qquad \text{when} \qquad \lambda = \lambda' = 1.$$

Thus along (31) lines of constant composition have the same properties as lines of constant entropy.

As for the ratio λ/λ' which occurs in the third equation of (29), it is easy to show that

$$\frac{\lambda}{\lambda'} < 1 \qquad \text{if} \qquad \left[\frac{\Delta H}{RT\Delta\nu} - \frac{\delta}{\delta - 1}\right]^2 > 0.$$

This relation holds throughout the (P, T) plane, except along (31).

Case III: $\Delta H\Delta\nu < 0$

Here ΔH and $\Delta\nu$ have opposite signs. While Case II is the most common, Case III is least common, and has some rather unorthodox properties. To begin with we note that λ and λ' may take on negative values, which suggests a complete reversal from the inert gas mixture behaviors. In particular, when

$$\frac{\psi\Delta H\Delta\nu}{RT} = -1 \tag{33}$$

λ and λ' become infinitely large. The meaning of (33) becomes evident if we substitute for λ and λ' in the third of (29), obtaining

$$\left(\frac{\partial P}{\partial V}\right)_s = -\frac{P}{V}\cdot\frac{C_P^*}{C_V^*}\frac{1}{1 + \Psi(\Delta\nu)^2},$$

$$= \gamma^*\left(\frac{\partial P}{\partial V}\right)_T, \qquad \gamma^* = \frac{C_P^*}{C_V^*}.$$

According to (21) $\gamma^* = 1$ when (33) is fulfilled. Thus Eq. 33 represents a line (most easily calculated as it occurs in the (P, T) plane) along which isentropics and isothermals are parallel. This is a marked departure from inert gas mixtures for which isentropics always cross isothermals at a finite angle. It is clear that in approaching and crossing

(33) the signs of λ and λ' change. It is interesting to examine the shape of (33). Consider, for example, the dissociation of ozone.

$$2O_3 \rightleftarrows 3O_2, \Delta H = -69{,}000 \text{ cal./mole } (0°\text{K}).$$

Here we take

$$\begin{aligned} \nu_1 &= 2, & \nu_3 &= 3, \\ \nu_2 &= 0, & \nu_4 &= 0, \end{aligned} \qquad \Delta\nu = 1,$$

$$\begin{aligned} n_1 &= 2(1-\epsilon), \\ n_3 &= 3\epsilon, \end{aligned} \qquad N = 2 + \epsilon.$$

The integration constants in (4) are necessarily zero. The mass action law becomes cubic in ϵ,

$$\frac{\epsilon^3}{(1-\epsilon)^2(2+\epsilon)} = \frac{4}{27}\cdot\frac{K}{P}, \tag{34}$$

while Ψ has the form

$$\Psi = \frac{\epsilon(1-\epsilon)}{6}.$$

To make possible a simple analytical treatment assume the heat of reaction to be constant. Thus van't Hoff's equation is integrated at once to give

$$\ln\frac{K}{K_o} = -\frac{\Delta H}{R}\left[\frac{1}{T} - \frac{1}{T_o}\right], \tag{35}$$

where K_o is the value of K for $T = T_o$. Having assumed ΔH constant (33) has the form

$$\frac{\Psi}{T} = -\frac{R}{\Delta H \Delta\nu} = \text{const.} \tag{36}$$

Now when Ψ is a maximum, $\Psi_{max} = \frac{1}{24}$, there is only one temperature, which we call T_o, which will satisfy (36). This is obviously the largest T-value which will satisfy (36). Thus

$$\frac{1}{24T_o} = -\frac{R}{\Delta H}$$

or

$$\Delta H = -24RT_o.$$

Consequently van't Hoff's equation becomes

$$\ln\frac{K}{K_o} = 24\left[\frac{T_o}{T} - 1\right]$$

or

$$K = K_o e^{24\left(\frac{1}{\tau}-1\right)},$$

where $\tau = \dfrac{T}{T_o}$ is introduced as the "reduced" temperature. Now when

$T = T_o$, there is only one pressure, $P = P_o$, for which $\Psi = \Psi_{max}$. And the maximum value of Ψ occurs when $\epsilon = \frac{1}{2}$. Substituting this information into the mass action law (34), we have

$$K_o = \frac{27}{20} P_o.$$

Thus

$$K = \frac{27}{20} P_o e^{24\left(\frac{1}{\tau}-1\right)}.$$

Substituting the last expression in (34) for K gives

$$\frac{\epsilon^3}{(1-\epsilon)^2(2+\epsilon)} = \frac{1}{5\pi} e^{24\left(\frac{1}{\tau}-1\right)}, \tag{37}$$

where $\pi = P/P_o$, the "reduced" pressure. Equation 37 gives ϵ as a function of π and τ. Substituting for Ψ and ΔH in (33)

$$\epsilon(1-\epsilon) = \tfrac{1}{4}\tau. \tag{38}$$

Equations 37 and 38 are a parametric representation of (33), ϵ being the parameter. For selected values of ϵ the values are given in Table I.

TABLE I.—*π and τ Values for Eq. 33.*

π	τ	ϵ
13.6	0.838	0.70
2.6	0.910	0.65
1.00	0.960	0.60
0.78	0.990	0.55
1.00	1.00	0.50
2.06	0.990	0.45
7.05	0.960	0.40
50.00	0.910	0.35

These values are plotted on the (π, τ) plane, Fig. 1.

From the first equation of (29)

$$\left(\frac{\partial \pi}{\partial \tau}\right)_S = \frac{\pi}{\tau}\frac{C_P}{R}\lambda$$

and we may now describe the general features of an isentropic in the (π, τ) plane. It is evident that:

(*a*) inside the region bounded by (33) $\left|\frac{\Psi \Delta H \Delta \nu}{RT}\right| > 1$, thus $\lambda < 0$,

(*b*) outside the region bounded by (33) $\left|\frac{\Psi \Delta H \Delta \nu}{RT}\right| < 1$, thus $\lambda > 0$,

(*c*) along (33) $\lambda = \infty$.

Thus an isentropic, which passes into the region bounded by (33), has the shape of line SS', crossing (33) at A and B. Note also that $\tau_A > \tau_B$. Clearly an isothermal drawn between τ_A and τ_B will cross the isentropic

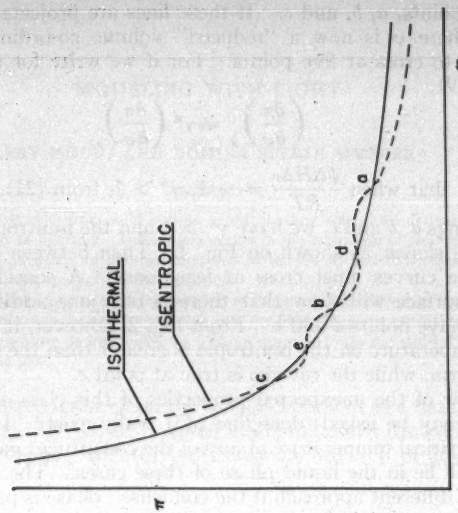

FIG. 2.

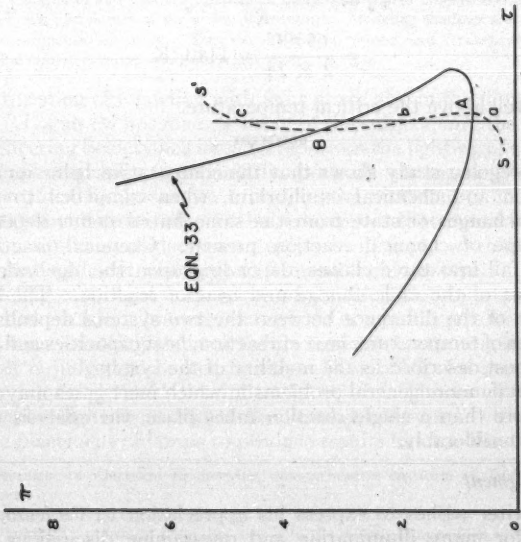

FIG. 1.

at three points, a, b, and c. If these lines are projected on the (π, φ) plane (where φ is now a "reduced" volume co-ordinate), they will be found to cross at five points. For if we write for the third equation of (29)

$$\left(\frac{\partial \pi}{\partial \varphi}\right)_s = \gamma^* \left(\frac{\partial \pi}{\partial \varphi}\right)_\tau$$

it is clear that when $\frac{\psi \Delta H \Delta \nu}{RT} \neq -1$, $\gamma^* > 1$, from (21). Therefore at the crossings a, b and c, we have $\gamma^* > 1$ and the isentropics have larger (negative) slopes, as shown on Fig. 2. Then between a and b, and b and c, the curves must cross at least once. A consideration of the (π, φ, τ) surface will show that there is only one additional crossing. Thus we have points d and e. From Fig. 2, however, it is clear that at d, the temperature on the isentropic is greater than the temperature of the isotherm, while the reverse is true at point e.

In view of the unexpected properties of this class of reactions the question may be asked; does line (33) really exist? If T_o is smaller than the critical temperature of any of the constituent gases then a part of (33) will lie in the liquid phase of these gases. The entire problem requires a different approach if the condensed phase is present. In the case of the ozone reaction

$$T_o = \frac{\Psi_{max}(-\Delta H)\Delta \nu}{R},$$

$$\approx \frac{69000}{2 \times 24} \approx 1440\ ^\circ\text{K},$$

which is well above the critical temperature.

SUMMARY

The foregoing study shows that the comparative behavior of fixed composition and chemical equilibrium, when compelled to undergo isentropic changes of state from the same initial point, depends first on the type of chemical reaction present. Chemical reactions are shown to fall into three classes, depending upon the algebraic sign of the product of the mole change and heat of reaction. The order of magnitude of the difference between the two systems depends on the magnitudes of temperature, heat of reaction, heat capacities and a fourth quantity best described as the mobility of the system.

To include more general problems in which inert gases may be present, or more than a single reaction takes place, the analysis must be extended considerably.

Acknowledgment

The writer wishes to express his appreciation to Professor H. W. Emmons for many illuminating and questioning discussions on this subject, and for his help in preparation of the manuscript.

MODELING WITH LIGHT

BY

PARRY MOON [1] AND DOMINA EBERLE SPENCER [2]

ABSTRACT

The problem of making solid objects appear as they are—three-dimensional—is largely a matter of lighting. The subject occurs in the education of children, in the illumination of statuary, in photography, and in television; and the treatment has been almost entirely qualitative and empirical.

This paper attempts to raise the problem to the quantitative level by introducing a numerical *modeling ratio* and by showing how the lighting can be predetermined by calculation to give any desired modeling, without recourse to the cut-and-try methods prevalent in the past.

In his extensive research on the school classroom as a hazard in child development (1),[3] Dr. D. B. Harmon has emphasized the importance of shadows in revealing the three-dimensional quality of objects. He says (2),

> Harsh modeling shadows seriously interfere with needed three-dimensional seeing experience. High modeling contrasts alter adversely the child's interpretation of the depth, form, size, and location in space of three-dimensional objects and tasks, and of the limits and nature of the artificial surround. Modeling shadows are necessary for three-dimensional seeing. They should be soft, graded, and "transparent" so as to define clearly contour, location, relationship, and size.

The question of modeling with light arises also in the illumination of statues. Logan (3) has made a study of the subject, and the Illuminating Engineering Society has issued a report on the lighting of art galleries (4). In photography, particularly portrait photography, sufficient gradation must be present to bring out the modeling of the face, but contrasts should ordinarily not be so extreme that detail is lost. In television, similar considerations apply or will apply when this new art has reached sufficient maturity to demand high quality.

Modeling with light is therefore of considerable importance. Most studies of the subject, however, have been of a purely qualitative nature—an arbitrary shifting about of light sources and reflecting screens until the result is judged to be satisfactory. The I.E.S., for instance, gives no data (4) on desirable variation in helios. The usual treatise on photographic lighting is also of the "cook-book type," with directions as to the placement of lamps to produce results like the illustrations but

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

with no quantitative information. Notable exceptions are the Mortensen books (5), which present specific recommendations.

In *Flash*, Mortensen states that in most cases he has found that a 4:1 helios ratio gives best pictures. That is, the helios of the highlights of a face should be approximately four times the helios in the shadows. This ratio gives transparent shadows, yet definite modeling. Harmon (2) finds from his experiments that the helios ratio should not be less than 3:1 nor more than 7:1. Thus these two authorities, working in quite different fields and with entirely different viewpoints, reach approximately the same conclusions.

In view of the wide applicability of modeling with light, it seemed advisable to make some additional experiments on helios ratios, and in particular to correlate these criteria with lighting methods that have recently become available. This is the purpose of the present paper.

CRITERIA

Matanovic (6) has advocated the use of a suspended sphere to indicate quality in room lighting. The sphere is painted a matt white, and its appearance, as hung in a room, gives an indication of whether the lighting is satisfactory. We propose to apply this idea to modeling.

Figure 1(*a*) shows how a golf ball appears when hung in a room with black floor, lighted with downlights. Figure 1(*b*) indicates the appearance with diffuse light from above and with a reflecting floor. The simplest criterion would seem to be the ratio of maximum helios to minimum helios for the portion of the sphere that is in the field of view. If the sphere is uniform and perfectly diffusing, this helios ratio is identical with the ratio between corresponding values of incident pharosage D_i. Thus the *modeling ratio* may be obtained as a ratio of helios readings or, perhaps more simply, as a ratio of values of D_i read from the familiar photoelectric "illumination meter." With symmetrical overhead lighting, the modeling ratio is obtainable as the quotient of two readings, one taken with the sensitive surface of the meter facing upward, the other with the surface downward.

We now establish a criterion for the modeling ratio. Evidently the exact value depends to some extent on aesthetic and psychological factors—a low ratio may be pleasing for the photography of children, a higher ratio may be desirable in obtaining a more dynamic or theatrical effect. It should be possible, however, to set a fairly definite lower limit, below which modeling is lost and a sphere begins to appear as a flat disk. It should be possible also to establish an upper limit, above which a normal observer feels that the contrast is unpleasantly high.

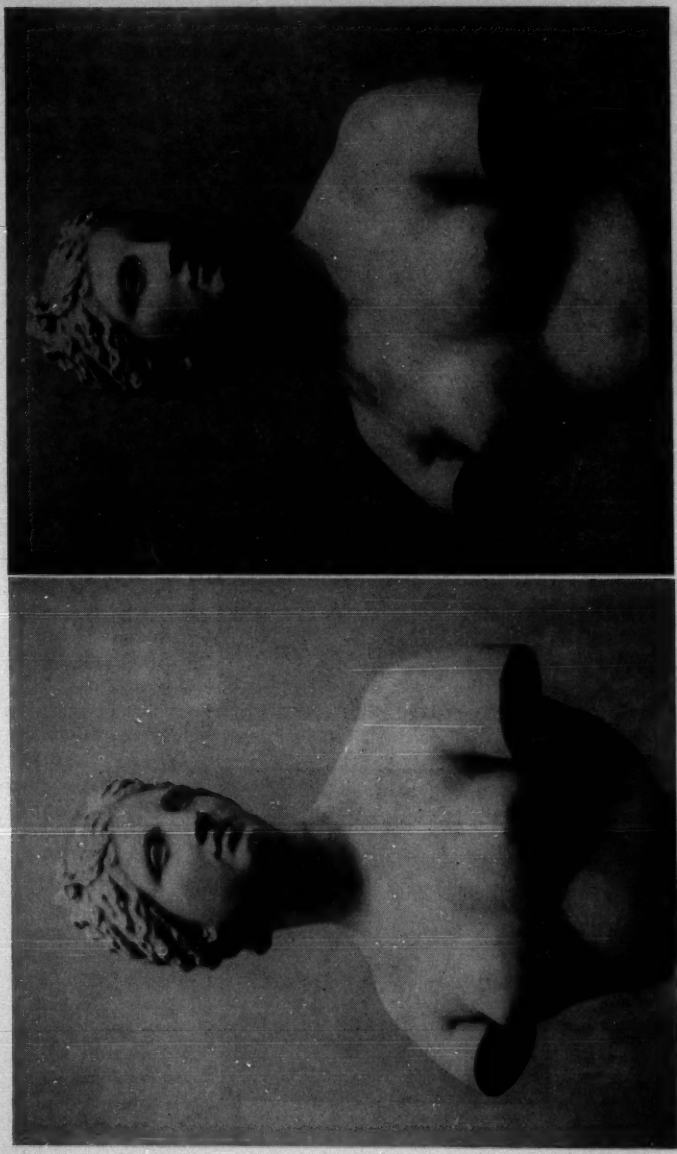
Our tests on golf balls and on statues indicate that the lower limit of the modeling ratio is approximately 2:1 while the upper limit is approximately 6:1. At a modeling ratio of 2:1, the object appears definitely three-dimensional, as shown in Fig. 1(*c*); but at 1.5:1, the

(a) With concentrated down-lights, dark floor. Modeling ratio, 100:1.

(b) With luminous ceiling and poorly-reflecting floor. Modeling ratio is approximately 10:1.

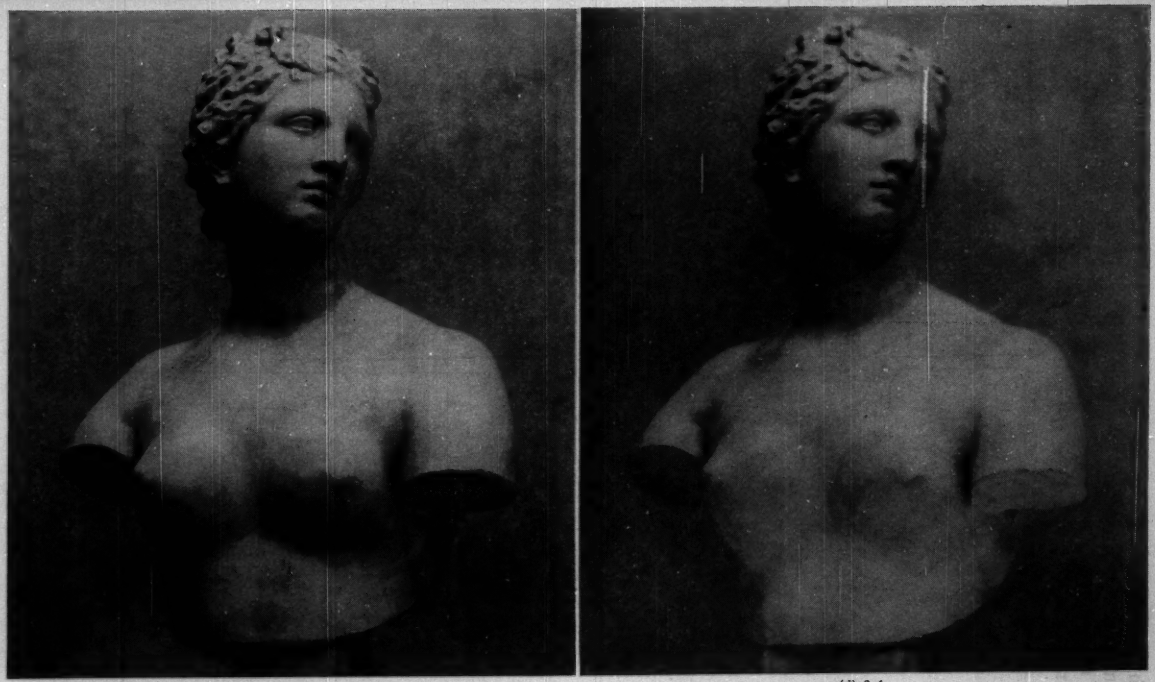
(c) With luminous ceiling and highly reflecting floor. Modeling ratio is approximately 2:1.

FIG. 1. Appearance of a golf ball, as affected by lighting.



(b) 5:1.

(a) 10:1.



(c) 3:1. (d) 2:1.

FIG. 2. Effect of changing the modeling ratio. Modeling ratios are shown under each photograph.

lighting is too "flat" and one begins to lose the sense of a three-dimensional object. On the other hand, a 10:1 ratio is definitely too contrasty for most purposes. We feel that 3:1 is best, though any value between 2 and 6 is allowable.

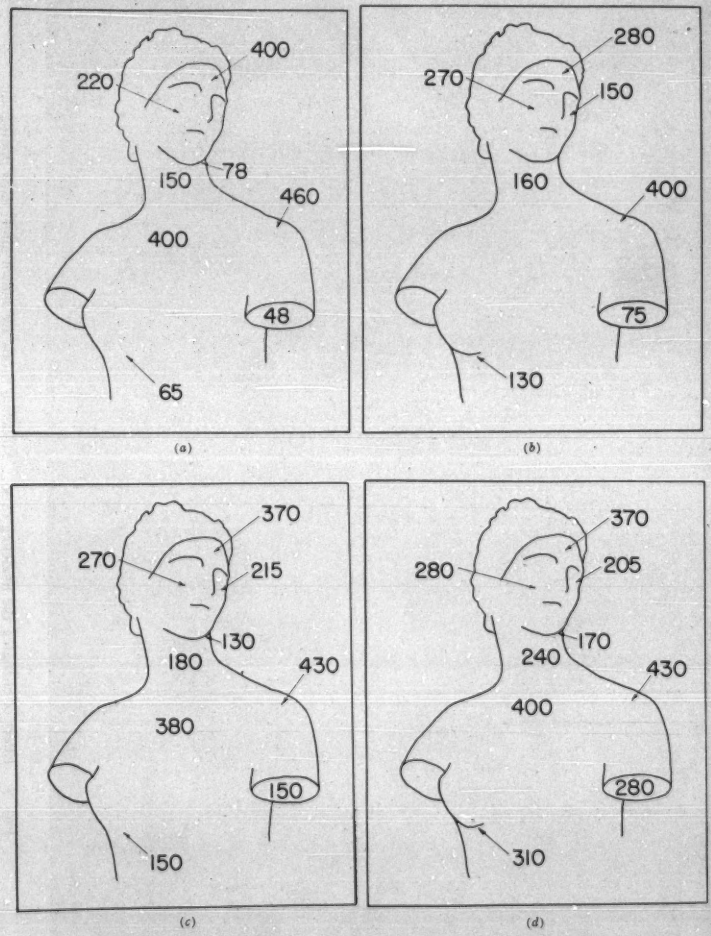


FIG. 3. Measured values of helios for the cases photographed in Fig. 2. The numbers represent helios in blondels.

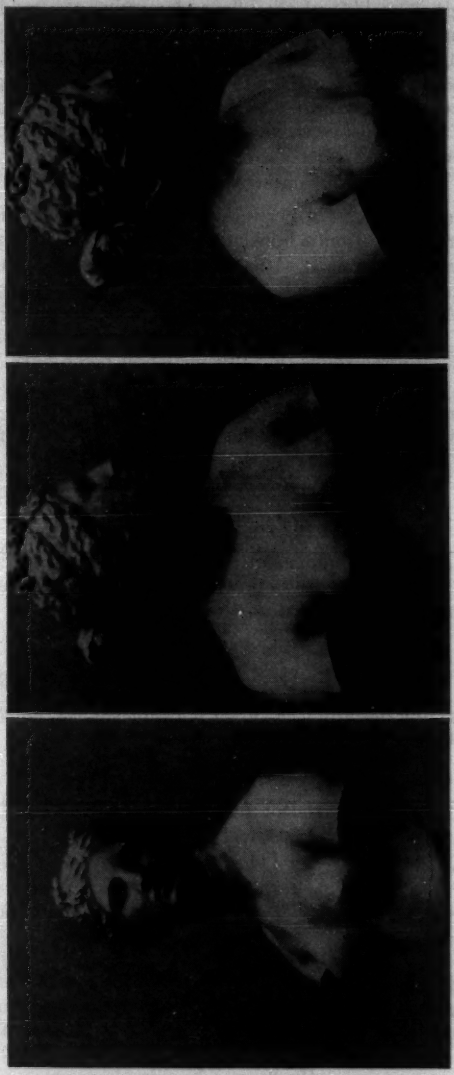


FIG. 4. Additional views with modeling ratio 3:1.

Figure 2 gives an idea of the effect of varying the modeling ratio. The room was lighted by means of a uniformly luminous ceiling of diffusing plastic (7); and the modeling ratio was changed by merely altering the reflectance of the floor, using black, white, or gray paper. Figure 3 gives helios measurements, as made with a Luckiesh-Taylor helios meter. Figure 4 shows some additional views with approximately 3:1 modeling ratio.

INFINITE ROOMS

After the establishment of a criterion for the best values of the modeling ratio, the next question is how can such values be obtained in practice. Undoubtedly the most satisfactory luminous environment (8) is produced by ceiling lighting, either by reflection (Type IIb) or by transmission (Type IIa). We shall now consider the modeling ratios obtained with Type II lighting. Because of the simplicity of the equations for the very large room ($k_r \to 0$), we shall begin with this special case and shall treat the finite room in the next section. The notation is the same as in previous papers (9) and in our book (10).

For the infinite room, the helios H_2 of the ceiling (including all interflections) and the helios H_3 of the floor are

$$H_2 = \frac{H_{02}}{1 - \rho_2\rho_3}, \qquad H_3 = \rho_3 H_2, \tag{1}$$

where ρ_2 and ρ_3 are reflectances of ceiling and floor, respectively (10). If a perfectly diffusing sphere is now hung anywhere in the room, the incident pharosage at any point on the spherical surface is

$$D(\theta) = \frac{H_2}{2}[(1 + \rho_3) + (1 - \rho_3)\cos\theta],$$

where θ represents the angle between a radius of the sphere and the vertical axis (upward). For the bottom of the sphere,

$$D(\pi) = \rho_3 H_2$$

so, for a sphere of uniform reflectance, the helios ratio is

$$\frac{H(\theta)}{H(\pi)} = \frac{D(\theta)}{D(\pi)} = \frac{1}{2\rho_3}[(1 + \rho_3) + (1 - \rho_3)\cos\theta]. \tag{2}$$

For the special case of $\theta = 0$, we have the modeling ratio:

$$\frac{H_{\max}}{H_{\min}} = \frac{H(0)}{H(\pi)} = \frac{1}{\rho_3}, \tag{2a}$$

and for the special case of the equator of the sphere,

$$\frac{H(\pi/2)}{H(\pi)} = \frac{1 + \rho_3}{2\rho_3}. \tag{2b}$$

By use of Eq. 2, the helios distribution is easily plotted for a uniform sphere hung anywhere in an infinite room. Figure 5 shows the distribution for floor reflectances of 0.10, 0.30, and 0.50.

In most cases, a consideration of the complete distribution on the surface of a sphere is unnecessary and we need find only the modeling ratio H_{max}/H_{min}. Figure 6 shows that if this ratio is to lie between 2

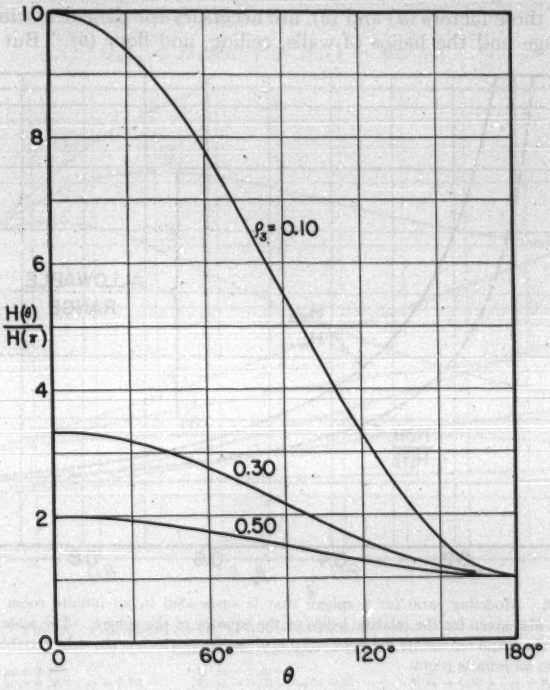


FIG. 5. Variation of helios over the surface of a sphere. The sphere is suspended anywhere in a very large room ($k_r \to 0$), and the distribution is affected only by the floor reflectance ρ_s.

and 6, the floor reflectance must be between 0.17 and 0.50. It is very seldom that an average floor reflectance exceeds 0.50 in practice, but there are many cases where values of 0.02 to 0.04 are employed. Our analysis shows, therefore, that even with a complete luminous ceiling in an infinite room, modeling will not be satisfactory unless the floor reflectance is at least 0.17. With small rooms, requirements are even more severe, as shown in the next section.

FINITE ROOMS

For finite rooms with ceiling lighting, the modeling ratio depends on

(*a*) Reflectances ρ_1, ρ_2, ρ_3,
(*b*) Domance k_r,
(*c*) Shape of the room,
(*d*) Position in the room.

Two of these factors (*a*) and (*b*), are necessary for the calculation of the pharosage and the helios of walls, ceiling, and floor (9). But in such

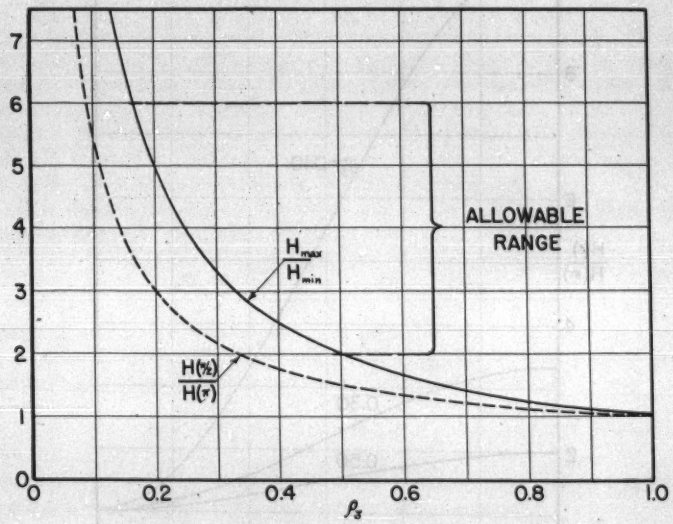


FIG. 6. Modeling ratio for a sphere that is suspended in an infinite room ($k_r \to 0$). A curve is also given for the relative helios at the equator of the sphere. The scale along the bottom of the graph represents the reflectance ρ_3 of the floor, which is the only variable affecting modeling in an infinite room.

calculations, the room shape is specified completely by the single factor,

$$k_r = \frac{S_1}{4S}, \tag{3}$$

where S_1 is the total area of the walls and S is the area of the floor.

The modeling ratio is, in a way, more complicated, since reflectances and domance are not sufficient to determine it exactly. In addition, one must decide on the point in the room at which he is interested in the modeling ratio. He must also fix the general shape of the room—circular, square, rectangular.

Perhaps the simplest calculations result by taking a circular room with point P on the axis, midway between floor and ceiling. Then, from Eq. 3, the domance is

$$k_r = \frac{h}{2a}, \tag{4}$$

where a is the radius of the room. The incident pharosage on a horizontal surface facing upward is

$$D_{max} = H_z \sin^2 \gamma + H_U(1 - \sin^2 \gamma),$$

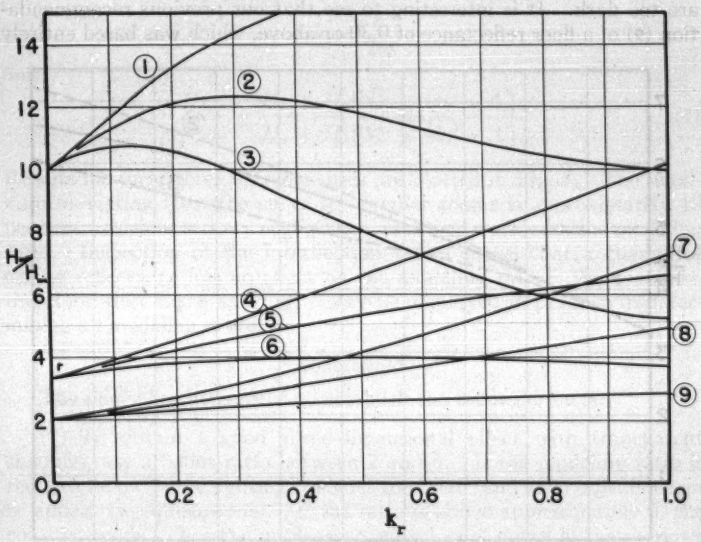


FIG. 7. Calculated curves of modeling ratio for circular rooms with ceiling reflectance $\rho_z = 0.80$.

1, $\rho_L = 0.30$; 2, $\rho_L = 0.50$; 3, $\rho_L = 0.80$ } $\rho_E = 0.10$

4, $\rho_L = 0.30$; 5, $\rho_L = 0.50$; 6, $\rho_L = 0.80$ } $\rho_E = 0.30$

7, $\rho_L = 0.30$; 8, $\rho_L = 0.50$; 9, $\rho_L = 0.80$ } $\rho_E = 0.50$

while on a surface facing downward

$$D_{min} = H_z \sin^2 \gamma + H_L(1 - \sin^2 \gamma),$$

where γ is the half-angle subtended at P by the ceiling or floor and

$$\sin^2 \gamma = \frac{1}{1 + k_r^2}.$$

Values of helios at the middle of upper and lower halves of the wall are

called H_U and H_L, respectively. Thus the modeling ratio is

$$\frac{H_{\max}}{H_{\min}} = \frac{D_{\max}}{D_{\min}} = \frac{H_2 + k_r^2 H_U}{H_3 + k_r^2 H_L}. \tag{5}$$

Some calculations made by use of Eq. 5 are plotted in Fig. 7. Note that the restrictions on the floor helios are even more severe than in the case of the infinite room. Figure 7 shows that in general the modeling ratio will be satisfactory for any floor reflectance between 0.30 and 0.50, but that lower reflectances will ordinarily give modeling shadows that are too dark. It is interesting to see that our previous recommendation (9) of a floor reflectance of 0.30 or above, which was based entirely

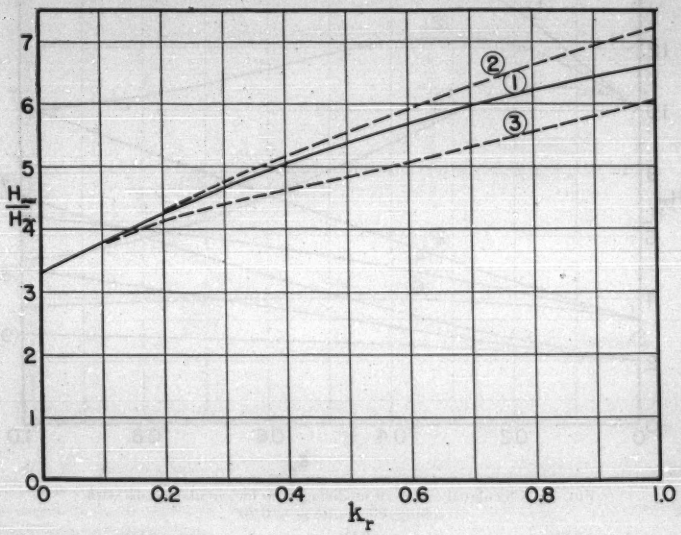


FIG. 8. Effect of room shape on modeling ratio, for $\rho_1 = 0.50$, $\rho_2 = 0.80$, $\rho_3 = 0.30$.

1, Circular rooms, $k_r = h/2a$.
2, Square rooms, $k_r = h/w$.
3, Narrow rooms of infinite length, $k_r = h/2w$.

on the question of getting proper helios distributions for good vision, turns out to be also a correct criterion for good modeling. The feeling is often expressed that ceiling lighting will give an uninteresting, flat effect; but both experiment and theory show this fear to be unfounded.

There remains the investigation of the effect of room shape. Two other extreme shapes were tried; the square room, and the narrow rectangular room of infinite length. With the square room, the horizontal

surface at P is lighted by four identical square panels representing ceiling or floor and eight identical rectangular panels representing the walls. The modeling ratio is

$$\frac{D_{max}}{D_{min}} = \frac{H_2(D_s/H) + 2H_U(D_y/H)}{H_3(D_s/H) + 2H_L(D_y/H)}, \tag{6}$$

where H_2, H_3, H_U, and H_L are obtained as before and where (D_s/H) and (D_y/H) are taken from the tables for the rectangular source (11).

With a rectangular room of height h, width w, and of infinite length,

$$k_r = \frac{h}{2w}$$

and

$$\frac{D_{max}}{D_{min}} = \frac{H_2 + H_U[\sqrt{1 + 4k_r^2} - 1]}{H_3 + H_L[\sqrt{1 + 4k_r^2} - 1]}. \tag{7}$$

Results for these three extreme cases are plotted in Fig. 8. The maximum deviation from the curve for circular rooms is approximately 10 per cent, which is entirely negligible in practical work with the modeling ratio. Inspection of the interflection tables shows that a change of ceiling reflectance has no effect on the modeling ratio. We conclude, therefore, that Eq. 5 and Fig. 7 may be employed in practice in determining all modeling ratios.

CONCLUSIONS

The paper has attempted to establish the following facts:

(1) To obtain a good three-dimensional effect with transparent shadows, use a helios ratio between 2 and 6. If the modeling ratio is reduced below 2, the lighting becomes too "flat" and solid objects begin to appear two-dimensional. If the ratio is above approximately 6, the contrast tends to become unpleasant, with loss of detail in the shadows or in the highlights.

(2) With ceiling lighting, the modeling ratio is affected mainly by the floor reflectance, which should be between 0.30 and 0.50, as indicated by Fig. 7.

(3) Our previous recommendation of ceiling lighting with $\rho_1 = 0.50$, $\rho_2 = 0.70$, $\rho_3 = 0.30$ not only satisfies the well-known 3:1 helios criterion for good vision (9) but it also provides excellent modeling.

These facts have important bearing on ideal lighting for schools, art museums, photographic studios, and television stations. For schools, correctly designed ceiling lighting, with the above recommended reflectances, produces an ideal visual environment that satisfies the 3:1 criterion. It also gives the pleasing three-dimensional effect emphasized

by Dr. Harmon (2). In the lighting of statuary, ceiling lighting and high reflectances are again desirable.

In portrait photography (12), in the production of moving pictures, and in television, the recommended system will always allow pleasing results. A wide variety of effects can be obtained, without any change in the lighting system, by merely using wall and floor coverings having several different reflectances. When more dynamic effects are desired, one luminous wall or one concentrated source, added to the luminous ceiling, will provide all the variety that can be needed, and will make impossible the common fault of several "dominant" directions of light and shadow.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

COMPRESSIBILITY STUDIES ON POLYMERS

Interesting information on the properties of polymers at high pressures—between 1000 and 10,000 atmospheres—has resulted from compression experiments[1] recently conducted by C. E. Weir of the National Bureau of Standards with the cooperation of the Carnegie Institution of Washington. Volume-pressure curves have been fitted to the experimental data, providing a convenient basis for comparison of compressibilities.

The high-pressure experiments were carried out as part of a broad program of research now under way at the Bureau on the fundamental properties of high polymers. Polymeric substances—such as rubber, plastics, textiles, leather, and papers—owe their strength, elasticity, and plasticity to the long, chain-like structure of their molecules. The Bureau is investigating the basic constants and properties of these materials in order to learn more about their molecular structure and to interpret their behavior in the form of useful products. While the high-pressure apparatus employed in this work was designed primarily for use in the Bureau's investigation of leather shrinkage, it has been found of value for studying the high-pressure properties of a number of high polymers.

In the experiments at the Bureau, a polymeric specimen was immersed at room temperature in a suitable pressure-transmitting liquid within the smooth bore of a thick-walled bomb of hardened steel. A leak-proof piston was then forced into the bomb, increasing the hydrostatic pressure within and thereby compressing the specimen. At intervals the depth of penetration of the piston and the corresponding hydrostatic pressure were measured. Depth of penetration was determined to 0.0001 inch by means of a dial gage mounted on the ram used to drive the piston into the vessel. Internal hydrostatic pressure was measured with a manganin pressure gage, which makes use of the very small change in the resistance of a coil of manganin wire with pressure. To take into account effects due to compression of the confining liquid, the piston and ram, the packing washers, and other parts, as well as distortion in the bore of the bomb, a comparison experiment was per-

* Communicated by the Director.

[1] For further details, see "High-Pressure Apparatus for Compressibility Studies and Its Application to Measurements in Leather and Collagen," by C. E. Weir, *J. Research NBS*, Vol. 45, p. 463 (1950) RP 2160; and "Compressibility of Natural and Synthetic High Polymers at High Pressures," by C. E. Weir, *J. Research NBS*, Vol. 46 (1951) RP 2192.

formed in which the specimen was replaced by a steel bar of similar volume.

For transmission of pressure to the specimen, a liquid was required which would not freeze in the pressure range studied and would maintain fairly low viscosity and electrical conductivity under high pressure. Only a few such liquids are known at present; many which ordinarily have very low viscosity become grease-like under extreme pressure. The use of Varsol for this purpose has thus far limited the materials investigated by the Bureau to those least likely to be affected by immersion in it. Polymeric materials studied include the synthetic plastics polyvinylidene chloride (Saran), polymonochlorotrifluoroethylene (Kel-F), polytetrafluoroethylene (Teflon), polyethylene, and a polyester (Selectron 5003); the raw synthetic oil-resistant rubbers Hycar OR-25, Hycar OR-15, Neoprene, and Thiokol ST; and the natural polymers cellulose and collagen (the principal constituent of leather). Several tanned leathers were also investigated.

In general, the compression observed for all the materials studied fell within a range between the compression values for true solids, which are rather low, and those of true liquids, which are considerably higher. This finding is in agreement with present concepts of the molecular structure of polymeric materials, which are believed to possess a higher degree of order than is found in liquids but less than that of true solids.

It was found that the compression of the specimens could be described within experimental error by means of the equation

$$-\frac{\Delta V}{V_0}=a(P-2000)+b(P-2000)^2+c(P-2000)^3$$

in which ΔV is the volume change between 2000 atmospheres (the base pressure in these studies) and the pressure P, and V_0 is the volume of

TABLE I.—*Compressibilities of Polymeric Materials.*

Material	*Coefficients of Pressure Terms*		
	$a \times 10^8$	$-b \times 10^{10}$	$c \times 10^{14}$
Polyvinylidene chloride (Saran)	2.17	19.0	8.73
Polymonochlorotrifluoroethylene (Kel-F)	0.99	4.74	0.83
Polyethylene	1.89	14.5	6.10
Polyester (Selectron 5003)	1.76	12.3	4.82
Hycar OR-25 (raw)	2.26	12.3	4.82
Hycar OR-15 (raw)	2.17	20.3	9.42
Neoprene (raw)	2.04	17.5	7.66
Thiokol ST (raw)	1.92	14.1	6.07
Cellulose (dry)	0.92	3.25	0.29
Leather (Collagen and various leather tannages)	1.23	5.60	1.35

the specimen at 1 atmosphere. The values of a, b, and c determined from the experimental data are given in Table I. The similarity in

the values of these coefficients emphasizes the close relationship between compressiblity and gross molecular structure, which is similar for all materials studied. Apparently small differences in molecular structure have little or no effect on compressibility; this is evident when the values for polyethylene, Neoprene, Thiokol ST, and Selectron 5003 are compared. The agreement between values for Selectron 5003—a brittle material—and the rubbers, which are highly elastic, emphasizes again the fact that the high elasticity of rubbers results from ease of deformation rather than high compressibility.

The lower compressibilities of cellulose, leather, and Kel-F were to be expected as these materials are somewhat denser at 1 atmosphere than the other polymers. The values given for leather represent an average of the data obtained on vegetable, chrome, and retan leathers as well as on hide collagen and kangaroo-tail tendon, all of which exhibit essentially the same compressibility. It was found that removal of the moisture ordinarily contained by leather (generally about 15 per cent by weight) had little noticeable effect on compression.

Many unexpected polymorphic transitions and freezing points have been encountered in the high pressure field. One of these, the freezing of water to form ice VI, occurs at 8710 atmospheres at 20° C. with a large decrease in volume. This transition provided a convenient reference point for calibration of the pressure gage used in the Bureau's experiments. In the measurements on the polymeric materials, the only transition observed occurred in Teflon (polytetrafluoroethylene) at 5500 atmospheres[2] and 25° C. and was accompanied by a 2-per cent decrease in volume. This abrupt change in compressibility made it impractical to fit the compressibility data for Teflon to an equation of the type given above. Below the transition the compressibility of Teflon is similar to that of Kel-F while at pressures above the transition it is nearer that of polyethylene.

Second-order transitions, observed as rather marked changes in slope of the volume-pressure curve, have been reported by investigators in other laboratories for some types of rubber in the pressure range studied. All of these rubbers, however, had been vulcanized and generally contained fillers. In the present studies no evidence of second-order transitions in the raw rubbers was found at room temperatures; all compression curves were smooth with the exception of that for Teflon.

ALASKAN ECLIPSE EXPEDITION

The National Bureau of Standards participated in an expedition to Attu Island, Alaska, to observe the total eclipse of the sun by the moon on September 11, 1950. The expedition was organized by Dr.

[2] This pressure is somewhat lower than that given by Bridgman in *Proc. Am. Acad. Arts and Sci.*, Vol. 76, p. 55 (1948).

John P. Hagen of the Naval Research Laboratory, primarily to apply the methods of radio astronomy to the study of the eclipse at various radio frequencies. Included in the party of 10 scientists were Grote Reber and E. A. Beck of the Bureau's Central Radio Propagation Laboratory.

In addition to infrared, visible, and ultraviolet radiation, the sun is continually giving off energy at radio frequencies. In recent years, the techniques of radio astronomy have been developed for receiving and analyzing these longer wavelengths. This has made it possible to obtain information about the positions of stellar bodies and the matter in interstellar space that could never have been seen with an optical telescope. As the radio waves given off by the sun are received without difficulty through rain or fog, the methods of radio astronomy proved particularly valuable for observations in the cool, damp climate of Attu Island, where the eclipse was accompanied by a severe rainstorm.

The path of the total eclipse began in the Arctic Ocean and proceeded southeastward across Siberia, the Bering Sea, and Attu, finally ending in the Pacific Ocean northwest of Hawaii. This path crossed United States territory only on Attu, a small piece of land, the last and most westerly of the Aleutian chain. While the climate of Attu is most undesirable for making visual observations with an optical telescope, it seemed likely that significant results could be obtained there with radio apparatus.

The party spent approximately a month on Attu installing equipment and making preparations. The center of the path of the eclipse crossed the eastern end of the island. However, this location is very inaccessible both by land and by sea. The apparatus was therefore mounted upon an old airplane runway on Alexai Peninsula, one of the few open flat places on the island. At the place of observation, which was about two miles from the center of the path of totality, the eclipse lasted 73 seconds.

The high-frequency energy from the sun was collected by a mirror 10 ft. in diameter having a focal length of 3 ft. An alti-azimuth mounting made it possible to sight the mirror on the sun. The mirror served to focus the incoming radiation on an antenna placed at its focal point and connected to a high-frequency receiver. Signals thus received were amplified and applied to an automatic recorder, providing charts of intensity versus time for various frequencies.

Measurements of solar radio intensity were made at wavelengths of 3 cm., 10 cm., and 65 cm. The receiving equipment for use at the 65-cm. wavelength, which was supplied and operated by the National Bureau of Standards, was a superheterodyne receiver having a 30-megacycle intermediate frequency and two stages of radio-frequency amplification. The measurements were made from two to four times a minute beginning about two hours before the eclipse. The sky at

an azimuth of about 90 degrees from the sun was used as zero reference. The sun was found to be quiescent and reasonably free from the transients which are often present during periods of solar activity.

Because of the radio waves sent out by the sun's ionized atmosphere, the apparent diameter of the sun at radio frequencies is always appreciably greater than at visual wavelengths. Consequently, all solar eclipses "viewed" with radio apparatus must appear to be annular rather than total. Preliminary results obtained from analysis of the data indicate that the apparent diameter of the sun exceeds that of the moon by 3, 7, and 11 per cent, respectively, at wavelengths of 3 cm., 10 cm., and 65 cm.

Besides the ionized atmosphere of the sun, a variety of other solar phenomena are effective in generating radiation at radio frequencies. For example, the magnetic field associated with sun spots causes additional radiation at centimeter wavelengths to originate in the neighborhood of the spots. Thus, as the edge of the moon first covered and later uncovered a spot group near the center of the solar disk, the observed intensity at radio frequencies dropped sharply and then rose again.

An eclipse of the sun by the moon, when viewed in an optical telescope, passes through four significant positions known as first, second, third, and fourth contact. First contact occurs when the edge of the moon appears just to touch the edge of the sun. When the moon covers the sun completely, second contact is said to occur. Third contact takes place when the sun again becomes visible, and fourth contact marks the instant at which the edge of the moon finally leaves the edge of the sun.

If the radio-frequency brilliance of the solar corona had been symmetrical, a minimum of observed intensity should have occurred at the time of optical totality, that is, between second and third contact. Actually, however, the minimum occurred a few minutes after totality at all three wavelengths. This was probably due to assymmetry of the corona caused by a group of spots near the east limb of the sun.

At first and fourth contact the apparent intensity rose about 10 per cent above that of the unobscured sun. The explanation of this effect is not yet clear; it may be due to reflections of solar energy from the surface of the moon at grazing incidence. Diffraction around the edge of the moon could not have been the cause as waves of kilometer rather than centimeter length would have been required. The possibility that this unexpected rise in intensity might have been caused by reflections from the landscape near the apparatus is now under investigation.

Industrial Supervisory Aptitude Tests.—A psychological test to predict how applicants for industrial supervisory positions will get along with those working under them as well as with the boss has been developed by University of Wisconsin psychologists. The test can be used by personnel administrators for selecting applicants for industrial supervisory positions or for selecting workers for promotion.

Former psychological tests of the sort used in selecting persons for supervisory jobs have been based on executive satisfaction with supervisors. This is the first test which tries to get at critical leadership qualities as well, since one of the critical factors in selecting supervisors is not only ability to carry out executive functions but to act as a group leader and keep workers satisfied.

The test, developed with the cooperation of management and workers of the Boston Store, Milwaukee, Wis., and the Milwaukee Electric company, consists of a dozen pages of multiple-choice questions. A competent psychologist or personnel worker can interpret the test and arrive at a good estimate of an individual's knowledge of jobs supervised, of technological controls applied to the jobs, of ability to plan and make decisions in applying management practices to work situations, and of leadership performance in dealing with supervised workers.

The results so far achieved indicate that questions of the sort used in the test can give "significant correlations" with executive performance, the psychologists report.

Gem-like sapphire laboratory crucibles developed by the Air Force have a compressive strength of 500,000 pounds per square inch, retain their strength at 2000° F. The crucibles, made of aluminum oxide, are polycrystalline structures of 98.8% purity, non-porous, and corrosive-resistant. Air Force engineers say the new material holds great promise for applications involving high temperatures necessary to melt super-hard metals, as alloys in making cutting tools to machine such metals, and in mixing with ceramics to produce jet turbine blades. The crucibles are produced through a process developed by Dr. E. Ryschkewitsch and Adolph Strott, former German scientists now employed by the Air Force.

A new cold weather boot developed by the Naval Clothing Depot features sealed-in insulation and should greatly reduce the number of frostbite cases which resulted from "Shoepac" use in Korea. The boot is rubber inside and out, with wool insulation sandwiched and sealed in as an interlining in the sole, tongue and around the foot and ankles. It is worn with only one pair of medium weight wool-cotton socks. Tests show that the boot not only reduces perspiration, but will even cause cold water coming in over its top to reach skin temperature within an hour. The Marines have contracted for large quantity production of the boot, which will eventually replace all "Shoepacs."

THE FRANKLIN INSTITUTE

MINUTES OF STATED MEETING

March 21, 1951

The March Stated Meeting of The Franklin Institute was held in the Lecture Hall on Wednesday evening, March 21, 1951.

The President called the meeting to order at 8:15 P.M. He reported that the Minutes of the Stated Monthly Meeting for February were printed in the March issue of the JOURNAL, and there being no corrections or additions they were approved as printed.

The Secretary reported the following elections to membership for the month of February:

Active	39
Associate	39
Student	6

The total Institute membership as of 28 February was 6,016.

The speaker of the evening, Dr. W. F. G. Swann, was introduced by the President. Dr. Swann, who is Director of the Bartol Research Foundation of The Franklin Institute, gave an interesting talk on "New Developments in Cosmic Rays." He explained the nature of the primary cosmic radiation, theories as to its origin, and the nature of the particles which comprise it. Illustrating his address with lantern slides, Dr. Swann discussed the bearing of intergalactic magnetic fields upon cosmic radiation, and described recent experiments having to do with the sun's magnetic field.

Dr. Swann was given a rising vote of thanks as the meeting adjourned.

HENRY B. ALLEN
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

KOTTLER, F.: The Goodness of Fit and the Distribution of Particle Sizes.

DANFORTH, W. E.: Thoria as a Cathode Emitter.

RABINS, LEONARD AND ERIC T. B. GROSS: Transient Analysis of Three-Phase Systems, Part II.

DRUCKER, D. C. AND F. EDELMAN: Some Extensions of Elementary Plasticity Theory.

EPSTEIN, HERMAN: Solution of Transients in Active Four-Terminal Networks.

MOON, PARRY AND DOMINA EBERLE SPENCER: Interflection Calculations for Various Luminaires.

WEINBERG, LOUIS: Solutions of Some Partial Differential Equations (With Tables).

MUSEUM

An interesting exhibit that might easily be overlooked by the visitor to the Museum is the replica of Joseph Saxton's magneto-electrical machine. This little apparatus was first shown before the members of the British Association for the Advancement of Science held at Cambridge in 1833, where it won the praise of such eminent scientists as Faraday, Wheatstone, Daniell (of battery fame), and the poet Samuel Taylor Coleridge.

Saxton had been a watchmaker in Philadelphia under Isaiah Lukens, but had gone to England to increase his knowledge of mechanics. Very shortly after his arrival in London, the bank where he had deposited his money failed leaving him friendless, moneyless, and jobless in a strange land. He obtained work at the Adelaide Gallery, a privately owned enterprise which had been founded in 1831 for the purpose of popularizing science. Saxton was engaged to construct exhibits that would provide entertainment as well as instruction. His first task was to build the "Paradoxical Head," an amusing hoax in which an expert swordsman was supposed to pass his sword through the neck of a dummy Turk's head without detaching the head from the body. This was accomplished by means of a series of complicated springs which held the head to the body. The passage of the sword-blade unlocked and re-locked these springs.

Among his other exhibits were a dynamometer for measuring the speed of a ship, a Perkin's steam gun, and a diving bell. None of these attracted the same admiration as his magneto-electrical machine. To appreciate its originality one must bear in mind that Michael Faraday and Joseph Henry had independently discovered magneto-electricity early in the year that Saxton made his machine, the presence of electricity produced by magnetism having been detected by means of a galvanometer.

For his inducing agent Saxton used a laminated permanent magnet made up of several strips of iron bent in the shape of a horse-shoe, securely fastened together. For armature he used a soft iron rod, ¾ in. in diameter, bent to the shape of a U, and around each arm he wound thirty or forty yards of insulated wire. By means of a simple crank driving a wheel and pulley, the armature was caused to rotate between the poles of the magnet. The little machine produced 3.5 volts at 100 r.p.m.

The replica on display in our Museum was made by Isaiah Lukens, the famous clockmaker, after Saxton's return to America. After nine years residence in London, Saxton's abilities had won for him high praise, and he had been offered the position of director of the printing machinery at the Bank of England. He declined this offer because of his ardent wish to return to Philadelphia, where his achievements had enhanced his reputation. In 1834, The Franklin Institute had awarded him the Scott Legacy Medal for his reflecting pyrometer. When he was offered the position of curator and constructor of weighing apparatus at the Philadelphia Mint, he eagerly embraced the opportunity to return home.

During his connection with the Mint, then located on North Seventh Street, he constructed the large standard balance for use in the annual essays and the verification of all standard weights of the government's assay coining offices. This balance may still be seen at the Philadelphia Mint. Saxton accompanied Alexander Dallas Bache to Washington when the latter was appointed Superintendent of the Geodetic Coast Survey, where he continued to invent apparatus and instruments. One of his most interesting ideas was the production of the first metallic cartridge case, to which he appears to have some worthy claim for priority. But he was very reluctant to take out patents for his inventions and omitted to do so on some ideas which would have brought him a good financial return.

He is best known for the reflecting pyrometer and for the automatic machine he devised for graduating the limbs of measuring instruments. But nothing else he did better illustrates his quick perception and his constructive skill, than does the little magneto-electrical machine.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS MARCH 21, 1951

ACTIVE FAMILY

Thomas F. Bayard, 3rd

ACTIVE

Thomas T. Andromidas
Harold W. Arndt
O. B. Arnold
C. Kenneth Baxter
Medford J. Brown
Edmond D. Crocheron
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St. Clair G. Gerdelmann
Paul C. Girard
Julius L. Gross
Ralph A. Huberman
George Truman Hunter, Ph.D.
Ludwig Kerst
Philip M. Lane
Robert A. Leary
Ernest N. May
John N. McDonnell, D.Sc.
Madeline Holland McDonnell, D.Sc.
William Smith Miller
Frank S. Moomey
Robert C. Moore
Harold Noskow
Robert S. Otter
H. H. Pakradooni, Jr.
Walter L. Schultze
Edwin J. Seiberlich
Benjamin H. Shoemaker, 3rd
Herbert J. Weeks

ACTIVE NON-RESIDENT

Donald W. Campbell
Lee Crow
Herman Jaffe
Robert L. Moore

NECROLOGY

Franklin Baker, Jr. '36
Melville G. Curtis '40
George F. Hoffman '43
Edward M. Hughes '36
Lawrence Morris '31
Eugene W. Wall '46
Fred M. Zeder '27

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

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BOOK REVIEWS

A Hundred Years of Physics, by William Wilson. 319 pages, illustrations and diagrams, 14 × 22 cm. London, Gerald Duckworth & Co., Ltd., 1950. Price, $3.50.

In 1946 Harvard's President, James B. Conant, delivered the Terry Lectures at Yale University and the text was subsequently published in book form entitled "Understanding Science." Because of the originality of viewpoint and the ability of President Conant to develop his ideas, this small book represents a significant utterance in clarifying the question of how the nonscientist may be given some appreciation of the tactics and strategy of science. Briefly, the theme of "Understanding Science" insists that the solution to the laymen's present bewilderment regarding science does not lie in a greater dissemination of scientific information among nonscientists. Rather, the scientific education of the layman is best achieved by the close study of relatively few historical examples of the development of science.

Care has been taken to explain this historical approach because William Wilson's "Hundred Years of Physics" is a striking example of the teaching method recommended by Conant. Wilson's book fulfills one fundamental requirement by limiting its coverage to a single century, from the beginning of Queen Victoria's reign to the present. Secondly, the historical approach is evident in the case history presentations which make for more interesting reading than can be found in most text books. The author discusses the cardinal discoveries of physics uncovered during the past hundred years and describes the growth of the theoretical structure by which these subjects were unified. Discoveries in thermal phenomena, statistical mechanics, light, electromagnetic theory, atomic structure, quantum mechanics and miscellaneous related subjects are highlighted with simple drawings, fundamental equations and references to their original publication.

The teaching properties of the book are weakened somewhat by the number of subjects covered and the resulting brevity. For instance, the Compton effect is treated in one-half page; Einstein's theory of the photo-electric effect, in two pages; the cyclotron, in three pages; the electron microscope, in a page. On the other hand, the writing is characterized by great clarity of expression typical of one who has taught the subjects for many years and crystallized his own understanding of the principles involved. William Wilson is Emeritus Professor of Physics in the University of London. His use of the first person strikes the American reader as a little unusual.

One strong impression gained from this book is a renewed confidence in man's current productivity of ideas. This confidence is gained from the preponderance of important work which the author points out as having been achieved during this past century. Such a point of view is in contrast to the opinion so often expressed implying that the truly great discoveries were mainly made long ago. This respect for the past stems partly from a hazy knowledge of dates and events. From this point of view, as well as reasons previously stated, the book is an interesting reference on modern physical principles and their origins. It is deserving of a place in any scholar's library.

C. W. Hargens

Laboratory Design, edited by H. S. Coleman. 393 pages, illustrations and diagrams, 22 × 30 cm. New York, Reinhold Publishing Corp., 1951. Price, $12.00.

The first report of the National Research Council's Committee on the Construction and Equipment of Chemical Laboratories, formed in 1924, was published in 1930 under the sponsorship of the Chemical Foundation. In 1947, under the name "Committee on Design, Construction and Equipment of Laboratories," a group representing industrial, institutional, and instructional laboratories started the preparation of a new and more inclusive report. The contributing authors include laboratory workers and administrators, consultants, and architects. The result is the present volume.

The book is divided into four main parts. Part I is concerned with Materials, Facilities, Services and Equipment, and includes an important section on Safety Precautions. Parts II and III, on Teaching Laboratories and Industrial Laboratories, provide information on location, design, and arrangements. Considerable discussion is directed towards the specific requirements of the many specialized laboratories and work spaces which are found in any teaching, research, or development organization, whether large or small. Part IV, Concise Descriptions of Some Modern Laboratories, contains detailed accounts of four industrial laboratories and nine college and institutional laboratories including a description of the Mellon Institute Research Laboratories by the Editor, Mr. Coleman, who is Assistant Director of the Mellon Institute.

While it is probably impossible for any single volume to furnish "all the answers," the completeness of this book transcends any estimate based on the size or number of pages, or the price. The style is clear, readable, and informative, never dogmatic. The photographs are attractive and the diagrams and architectural plans clearly drawn and, fortunately, large enough in reproduction for easy and detailed examination. The value of the book is enhanced by a selected bibliography and an index.

The book will be of interest to anyone concerned with laboratories and their work, whether involved in minor modifications to improve efficiency, safety and comfort or in the planning of new installations or entire new laboratory organizations.

C. T. CHASE

FUNDAMENTALS OF QUANTUM MECHANICS, by Enrico Persico, translated and edited by Georges M. Temmer. 484 pages, diagrams, 15 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $6.00.

Here is a book for which the author, translator and publisher should be congratulated. It is practically three compact books in one. The author has chosen to separate the intuitive approach to the subject from the abstract and formal approach instead of intermingling them as is common in other introductory treatments. In the first part, covering some 80 pages, one gets an over-all view of the evolution of quantum mechanics with descriptions of important experiments which led to its development, in the manner of elementary books on atomic physics. The second part, about 200 pages, covers the wave mechanics of a particle based on the uncertainty principle. This part contains a mathematical introduction concerned with differential equations, expansion of functions in a series of eigenfunctions, in a Fourier integral and the wave interpretation of the Fourier expansion. The concluding chapter of this part shows the older Bohr-Sommerfeld theory as a first approximation of the wave mechanics and treats some of the important results of that theory. Finally, in the third part, again some 200 pages, the author presents the principles of quantum mechanics in the more abstract form of transformation theory using more advanced mathematical methods, such as Hilbert space and matrices, which are explained in a mathematical introduction.

The chapters on mathematics are undoubtedly too brief and should be supplemented by separate courses in mathematics. The only application of the methods is to the theory of the hydrogen spectrum in the second part of the book. The reviewer feels that potential physicists, chemists and physical chemists in general could profitably begin the study of quantum mechanics with this book. To have an understanding of the origin, evolution and implications of quantum mechanics before plunging into the more abstract treatments and applications, is believed to be important for most students in the fields of chemistry and physics, for there are few Sommerfelds among them, or even Paulings. A study of this book can be followed by others of a still more formal nature or by those treating in detail approximation methods and applications.

The book is clearly written, well organized and has a clean look about it. In the translation all physical constants were replaced by their most recent values and some additional notes and corrections supplied by both the author and the translator. The bibliography contains principally references to related books on quantum mechanics and mathematics.

W. E. SCOTT

INDUSTRIAL AND SAFETY PROBLEMS OF NUCLEAR TECHNOLOGY, edited by Morris H. Shamos and Sidney G. Roth. 368 pages, illustrations and diagrams, 14 × 21 cm. New York, Harper & Brothers, 1950. Price, $4.00.

Eighteen papers presented during a three-day conference sponsored jointly by the Atomic Energy Commission and New York University's division of General Education are presented in this book. The papers are concerned with the non-military and constructive aspects of nuclear technology, and are presented in language readily understood by the layman. Both of these factors make the book of interest to great numbers of readers having varied backgrounds and interests in nuclear science.

Part I covers activities of the United States Atomic Energy Commission. In this part are discussed aspects of the future of atomic energy; the status of current non-military atomic energy developments (a review of "who is doing what" in such development); contracts with the AEC (the structure and policies of the AEC, the nature of contracts issued by it, and information on contracting with AEC); and the status of inventions and inventors under contracts of the AEC.

Part II is concerned with radiochemistry and isotopes and contains very interesting papers on the applications of isotopes in such fields as biology, medicine, plant growth, chemistry, measurements, etc. Several papers giving useful background information on the fundamentals of nuclear energy and radiation, electronuclear apparatus, and radioactive materials are also included in Part II.

In Part III are discussed the problems in connection with the construction of and equipping of radiochemical laboratories. Typical examples from past experience are cited in this part, which also contains a number of photographs demonstrating laboratory techniques.

Hazards, Safety, and Insurance is the title of Part IV, in which emphasis is placed on the intelligent control of the hazards associated with nuclear work. The hazards to be encountered are discussed as well as the methods used to control or avoid the harmful effects of such hazards.

This book is an enlightening piece of work which brings together in a single volume many of the facets of nuclear science. The level on which it is written commends it to the general public as well as to those engaged in scientific work.

ROBERT S. GRUBMEYER

ELECTROMAGNETIC WAVES AND RADIATING SYSTEMS, by Edward C. Jordan. 710 pages, diagrams, 14 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $7.75.

Professor Jordan has written a textbook on electromagnetic waves which, according to this reviewer, is unequalled for its purpose. The volume is intended to give mature graduate students in communications and electronic engineering and physics a clear understanding of the role electromagnetic waves play in a modern communications system. It is different from many other texts treating this subject because the fundamental relationships of the electromagnetic field are translated into the solution of engineering problems. Practicing communications and electronics engineers today are well cognizant of the limitations of ordinary circuit theory and the need for field theory to bridge an existent gap.

The author has been most efficient in his use of material and words for each of the seventeen chapters is informative and basic. Since the mks system of units is employed it was deemed more instructive for all developments of electromagnetic relationships to start from fundamentals. A review of vector analysis, co-ordinate systems, and definitions is followed by chapters devoted to electrostatics and the steady magnetic field. For each item discussed, the author presents the experimental basis of the well known relationships. Armed with these elements, the reader next finds chapters on electromagnetic waves, power, waves in simple structures, transmission lines and guides, radiation and impedance relationships for a wide variety of antennas, secondary sources, and apertures; and ground wave propagation. Each chapter closes with a set of problems, making the book even more suitable for classroom use.

Applicative engineering problems are always considered, either by pointing out limitations to the theory or by actual illustrative computations. Examples of this are the sections devoted to: transmission line stubs and matching sections; means for exciting wave guides; impedance

characteristics of antenna structures; polarization effects; antenna synthesis; and the detailed design of a variety of antenna systems including slits, slots and horns.

While the book may have been published too late for classroom adoption in 1950-1951, this reviewer is of the opinion that it may find wide use in the near future for either a one- or two-semester course at the graduate level. Needless to say, many practicing engineers will not only refer to this book for new material but they may wish to purchase it as a permanent investment.

S. CHARP

COSMIC RAYS, by Louis LePrince-Ringuet, translated by Fay Ajzenberg. 290 pages, illustrations, 15 × 22 cm. New York, Prentice-Hall, Inc., 1950. Price, $5.00.

The subject of cosmic rays although very interesting and fairly old is still not completely understood at the present time. In this book the author brings the reader up to date by presenting him with a clear and accurate discussion of the present state of cosmic ray research.

The book consists of seventeen chapters. The first two of these deal with some of the fundamental concepts of atomic physics which must be understood before undertaking a study of cosmic rays. In the third chapter considerable space is devoted to a discussion of the methods used in the detection and study of cosmic rays.

Among the topics discussed in the remaining chapters are: the characteristics of cosmic rays at various altitudes, the effect of latitude and the earth's magnetic field on cosmic radiation, meson theory, the results of high-altitude research, and the origin of cosmic rays.

The author includes many excellent photographs which amplify the material discussed in the text. The book is written in a descriptive manner and contains no difficult mathematics. It should be of particular interest to those in other fields who wish to increase their knowledge of this fascinating subject.

M. D. EARLE

OPERATIONAL CALCULUS BASED ON THE TWO-SIDED LAPLACE INTEGRAL, by Balth van der Pol and H. Bremmer. 415 pages, 16 × 26 cm. New York, Cambridge University Press, 1950. Price, $10.00.

The authors state that this book is intended for application of the operational calculus in its modern form to mathematics, physics and technical problems. There are a number of books available on the theory of the Laplace transformation and its applications to various practical problems, but in this book the treatment is based on the two-sided Laplace integral, contrary to the usual practice based on the one-sided integral. The two-sided Laplace integral has its lower limit of integration equal to $-\infty$ instead of 0. The class of functions suited to operational treatment becomes larger and transformation rules are simplified by this approach. The starting point is the Fourier integral from which the Laplace integral is derived. A detailed discussion is given of Heaviside's unit function and the delta or impulse function also used by Heaviside. Questions of convergence in connection with the summary of series and integrals are investigated in detail. Other chapters are concerned with the operational treatment of differential equations with constant coefficients, including the theory of linear electrical networks and transient phenomena, with differential equations having variable coefficients including applications to Legendre polynomials, Bessel functions, etc. Study is made of step functions with applications to number theory. The operational calculus is applied to difference and integral equations. Applications of the theory to problems in several independent variables is covered in two chapters. The final chapter is concerned with the simultaneous transformation of more than one variable, leading to a simultaneous operational calculus.

To make the material more readily available for application the authors have listed the most important results at the end of the book. One list called the "grammar" contains the operational rules and a second list called the "dictionary" is intended to be helpful in translating the language of one variable to that of another.

The authors feel that the book will be of interest to both pure mathematician and practical man. The reviewer feels that the authors might have written two books although it must be admitted that the subject matter has been clearly designated in the introduction, so that these two groups of readers may select the material of interest with a minimum of effort.

W. E. SCOTT

ION EXCHANGE RESINS, by Robert Kunin and Robert J. Myers. 212 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $4.75.

This book is a clearly written, workmanlike account of the rather recent field of ion exchange resins. These resins have built into them many accessible ionizing groups, so that the parent resin molecule represents one giant ion of one sign, and the ionizing groups provide, when in contact with solution, a flock of soluble ions of the opposite sign. Since the resin is not very particular as to the nature of its attendant soluble ions, if when it is stocked with one kind it is exposed to a solution containing another it will exchange. These resins, and their analogues among earlier known, naturally occurring, clays, silicates, etc., are finding wider and wider usage as water softeners, analytical tools, and for many other purposes. The authors have thus done a valuable service in bringing together for the first time the already extensive literature on the theory, the preparation and the uses of these systems.

The book is essentially an engineering handbook for the worker in this field. The first few chapters present somewhat briefly and uncritically theoretical ideas as to the mechanism of ion exchange, and the physical and chemical natures of these resins. A chapter is devoted to the synthesis of the commonly used types, and then the rest of the twelve chapters is given over to applications. Included here are fractionation of electrolytes, water softening and deionizing, analytical applications, and a few others. Methods of studying the chemical and physical properties are discussed, and a chapter is devoted to the design of ion exchange units.

All of the practical section of the book is marked by generous use of charts and tables of valuable engineering data. An imposing bibliography of 615 references is included, and an appendix containing table of specific gravities of common solutions, etc.

A. D. FRANKLIN

BOOK NOTES

TRANSFORMER ENGINEERING, by L. F. Blume and others. Second edition, 500 pages, diagrams, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1951. Price, $7.50.

The authors have presented a detailed study of transformer performance aimed to aid the engineer solve technical problems involved in the operation of transformers. The second edition is not only a revision of material, but has been rewritten to include recent advances made in the field, particularly those relating to insulation, thermal characteristics and load ratio control practice. The book is one of the General Electric Series written by GE authors for the advancement of engineering practice.

ALTERNATING CURRENT CIRCUITS, by Russell M. Kerchner and George F. Corcoran. Third edition, 598 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1951. Price, $5.50.

The fundamentals of alternating current circuits, both theoretical and applied, are presented including an introduction to network analysis, the basic concepts of the attenuation and phase shift, and the method of designing double tuned circuits.

This third edition contains information on nodal method of circuit analysis, a method of designing tuned coupled circuits to have specified decibel variation over a specified band width, the three-origin vector diagram of a polyphase circuit and material on the Q of electrical circuits, which has been revised. Illustrative oscillograms of actual circuit performance and problems for which answers are given make it a valuable text for use of both student and teacher.

DISTILLATION EQUILIBRIUM DATA, by Ju Chin Chu. 304 pages, 16 × 24 cm. New York, Reinhold Publishing Corp., 1950. Price, $6.00.

A collection of vapor-liquid equilibrium data on binary, ternary and multiple component systems—a total of 176 systems. The data are tabulated, together with temperature in degrees Centigrade and pressure in millimeters of Hg in alphabetical order of the common name of the component which is most volatile in its pure state. A system number is assigned, a complete chemical formula index and original references to the data are listed. An exceptional reference book to information needed in design, performance prediction and operating characteristics of distillation equipment.

BORDERLANDS OF SCIENCE, by Alfred Still. 424 pages, 14 × 22 cm. New York, Philosophical Library, 1950. Price, $3.75.

This book is an attempt to evaluate the influence on civilization of both science and superstition. The lives and work of men like Paracelsus, Agrippa, Copernicus, Fracastoro, Cardan, Ramus, Galileo, Kepler, Newton are presented. In the fourteen chapters science and magic, witchcraft, the divining rod, levitation, hypnotism and time are covered. The book contains a fine author and subject index and is a well documented work.

PHOTONS AND ELECTRONS, by K. H. Spring. 108 pages, diagrams, 11 × 17 cm. London, Methuen & Co. and New York, John Wiley & Sons, Inc., 1950. Price, $1.75.

A brief account of the main way in which electrons interact with radiation, particularly emphasizing phenomena associated with higher energies. Methods have been subordinated to results. Three processes causing the emission of electrons by photons are discussed, namely, the photo-electric effect, the Compton effect, and the electron-pair production. The final chapter gives a general survey of the cosmic ray phenomena. A list of references is appended as a guide to the reader. The work is one of the Methuen Monographs on Physical Subjects.

GAS PRODUCERS AND BLAST FURNACES, by Wilhelm Gumz. 316 pages, diagrams, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $7.00.

Previous literature on the subject of gasification and its allied subjects has largely been confined to the empirical approach. Gumz, realizing that an evaluation of these empirical data is needed for a better understanding of gas technology, presents herein such an evaluation, complete with calculations. He emphasizes the theory with accompanying calculations, believing that by so doing, much costly experimenting can be eliminated—since calculation is more rapid and decisive than experiment. Theory is illustrated by well-chosen practical examples, in which the author proves the generally close agreement between calculation and observation. The book is a welcome addition to the library of the designer of gas production equipment and to the engineer concerned with problems of gas technology.

REVIEW OF TEXTILE PROGRESS, edited by W. J. Hall and C. W. Harper. Volume 1, 1949. 349 pages, 14 × 21 cm. Manchester, The Textile Institute and the Society of Dyers and Colourists, 1950. Price, 1 pound.

The Textile Institute and the Society of Dyers and Colourists have jointly published Volume 1 of "Review of Textile Progress" relating to progress made in 1949. It has been planned to be a comprehensive coverage made annually in all fields of textile technology. In instances where progress has been limited, these sections will be dealt with at intervals of two or three years. Experts have compiled the sections separately, Volume 1 containing sections on: the chemistry of natural and man-made fibres; bleaching, dyeing and printing; finishing of wool, silk and rayon; processing; analysis, testing, grading and defects; laundering and dry cleaning; and textile machinery.

AN INTRODUCTION TO THE SCIENCE OF PHOTOGRAPHY, by Katherine Chamberlain. 292 pages, illustrations, 16 × 24 cm. New York, The Macmillan Company, 1951. Price, $4.75.

The purpose of this book is primarily to provide the amateur photographer with a chemical and physical background necessary for an understanding of the processes involved in photography—from the processing of films, through optics, and ending with the art of color photography. An introductory history of photography includes a chronological table of important events from 1568 through 1906. The main body of the book is devoted to a discussion of the problems, the processes and theories involved therein, and the solutions thereto, encountered by amateur photographers. Each chapter contains a list of questions and problems, as well as suggested additional reading material. An interesting feature is the section at the end of the book in which are found the answers to the problems set forth in the individual chapters. Miss Chamberlain is to be congratulated upon her fine presentation—it more than adequately fulfills its objectives. The book is recommended to all amateur photographers who wish to improve their understanding and practice of the art.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Alteration of Biological Material by the Waring Blendor.—GEORGE M. NAIMARK AND WILLIAM A. MOSHER. The Waring Blendor and similar blender-liquidizers have proven valuable in biological laboratories as rapid, efficient instruments for emulsification, disintegration and comminution. The high-speed (10,000 to 20,000 rpm.) rotation of the cutting assembly of such blenders quickly reduces many substances to homogeneous, finely-divided slurries. Recently, however, various workers have reported the existence of undesirable chemical and physical effects as a result of blender treatment.

(1) *Heat Generated During Blending:* The revolution of the blender motor at such high speed generates a great deal of heat in the material undergoing treatment. As a result care must be taken to control the temperature when working with heat-labile biological substances. Stern and Bird (3) reported a rise in temperature of 18°C. when treating a 250 gram suspension for 5 minutes and Lamden (1) found an average temperature rise of 8.7°C. when blending 200 ml. portions for 3 minutes. The following table shows the rises in temperature as a result of blending a 0.2 per cent pepsin solution for 1 minute.

Duration of blending	Size of sample treated	Temperature before treatment	Temperature after treatment
1 min.	100 ml.	19.8°C.	27.5°C.
1 min.	100 ml.	19.5°C.	28.0°C.

Average temperature rise as a result of treating 100 ml. of the enzyme solution for 1 minute was 8.1°C. Hence, temperature rise within blender containers must be taken into consideration when dealing with biological substances which are damaged by heat.

(2) *Surface Denaturation of Proteins:* The rapid formation and destruction of surfaces caused by foaming while blending leads to surface denaturation of protein materials. In addition, cavitation as a result of the rapid movement of the blender blades through the solution undoubtedly contributes to the surface denaturation effects of blending.

Such surface denaturation may be readily detected by studying the effects of blender treatment upon enzymes. Since enzymes are protein in nature they may be denatured by the action of surface forces. A decrease in enzyme activity follows denaturation and may be detected by the use of standard procedures. Quinlan-Watson and Dewey (2) reported inactivation of the enzyme cytochrome oxidase as a result of blending and Stern and Bird (3) found that the dehydrogenase activity of wheat germ extracts was reduced as was the ascorbic acid oxidase activity of wheat flour suspensions.

In our laboratory it was found that a 0.2 per cent solution of U.S.P. Merck Pepsin was almost completely inactivated by treatment in a blender for 1 minute. In preparations of biologicals utilizing blenders one should take into account the denaturing effects of even short periods of blender treatment.

(3) *Oxidation of the Blended Substance:* The vigorous mixing caused by the high speed of the motor introduces large quantities of air into the substance undergoing treatment often causing oxidation. Such intimate contact is established between oxygen and the substance being treated that oxidation is to be expected where possible. Stern and Bird (3) reported oxidation of sulfhydryl groups as a result of blending. This difficulty may be obviated by the use of an atmosphere of inert gas above material being treated.

(4) *Copper Contamination from Worn Blender Parts:* It has been recently pointed out by Lamden (1) that blender parts may readily be a source of copper contamination when underlying brass is exposed by worn chrome plating. Such copper contamination has been shown to cause oxidative loss of ascorbic acid and possible poisoning of enzyme systems. Care must be exercised to use blenders with well-plated parts when traces of copper may introduce errors.

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(2) QUINLAN-WATSON, T. A. F., AND DEWEY, D. W., *Australian J. Sci. Research* B, 1: 139 (1948).
(3) STERN, R., AND BIRD, L. H., *Biochem. J.*, 44: 635 (1949).

***Abstract of* Effect of Adenine on Rats.**—J. O. ELY AND M. H. ROSS (*Arch. Path.* 50, 359 (1950)). Adenine was fed at various levels to rats for different periods. A dietary level of 0.17 per cent adenine caused some enlargement of the kidneys and reduction of the size of the liver and the thymus. The maximum level that had no influence on growth was 0.33 per cent of the diet (261 mg. per kilogram of rat weight per day). Higher levels reduced growth in addition to causing enlargement of the kidneys and reduction of the size of the liver and the thymus.

Crystalline deposits, probably uric acid, were observed in renal tubules of the rats which had been fed adenine. The amount of alkaline phosphatase in the proximal convoluted tubules of the kidney was greatly reduced as a result of adenine feeding, and thymic involution occurred.

Doses of adenylic acid corresponding in adenine content to the doses of adenine that caused great enlargement of the kidneys, involution of the thymus and decrease of the size of the liver failed to elicit any of these effects.

CURRENT TOPICS

Lead-Calcium Storage Batteries.—A new, long-life storage battery, requiring much less maintenance and lasting at least half again as long as those now in use, has been developed by Bell Telephone Laboratories scientists for use in telephone central offices throughout the nation.

The battery was developed for the specific requirements of the telephone industry. It is not, at present, recommended for other uses. As it becomes commercially available and more information as to its characteristics is obtained, its field of usefulness may be considerably extended.

The new battery, which is less subject to corrosion, has the further advantage of serving for many months without the addition of water. And it loses only 4 per cent of its total charge each month—one-fifth of the amount lost by present storage batteries.

Since it does not often have to be replaced, and requires so little maintenance, the battery is expected to reduce considerably the cost of providing standby power for the nation's telephone network. In the Bell System alone there are a million storage battery cells connected to telephone circuits. In the rare event of power failure, these batteries are ready to supply the vital current for telephone service. Current rarely flows in or out of these cells, beyond the tiny trickle which keeps them charged. But even in this standby service, the batteries require water. They also consume power, and eventually they wear out.

By changing a single ingredient, the clue to which came unexpectedly from another line of research, the Bell scientists expect to achieve a saving of more than a million dollars a year. Production of the new batteries has already been started. As they become available, they will gradually replace the present type of battery, which has been in use for many years.

A minute trace of an unusual gas in a telephone office started the Laboratories' chemists on the road to their significant discovery. They were searching for atmospheric impurities which they suspected might be tarnishing parts of the elaborate switching mechanism used in the dial system. They carried their investigation to all parts of the telephone central office, eventually setting up their experiments in the battery rooms. Here they detected in the atmosphere, during the overcharge period, a gas having a very different test reaction from any they had found elsewhere. Uncertain at first of its identity, they returned to the Laboratories, set up some storage batteries, overcharged them and collected enough of the unknown gas by freezing it out in a liquid air trap. The frozen gas looked like a small pile of snow, and on analysis it proved to be antimony hydride, commonly known as stibine. Strangely enough, it was not stibine which was causing the tarnishing of equipment, but detecting its presence gave the chemists an important clue to some peculiar goings-on within the batteries.

After exhaustive research they found that antimony, a metal commonly used to harden lead, was passing undetected from one plate to another within the cell, and was speeding up corrosion and causing an electro-chemical

action which resulted in partial discharge of the negative plate. In this process, the by-product, stibine gas, was being produced. It seemed clear, then, that if they could eliminate the antimony, a better battery would result.

Searching for a way to do this, they turned to another field of telephone research. They learned that in experimenting with hardening agents for the lead covering of cables, Bell Laboratories metallurgists had successfully used small amounts of calcium instead of the usual antimony. Less than 0.10 per cent of calcium, compared with the 12 per cent of antimony usually employed in battery alloy, could do the job. And the battery chemists found, too, that calcium does not cause any of the troubles for which antimony had been responsible.

From here it was only a brief step to development of the new, lead-calcium storage battery, although it has taken several years to prove its suitability and endurance under actual operating conditions.

Advanced Time Scale Analog Computer.—A new type of electronic brain for solving problems relative to the design of automatic machinery has been developed by The Franklin Institute Laboratories for Research and Development. Known as the Advanced Time Scale Analog Computer, the instrument is an electronic model of complicated mechanical devices such as automatic machinery used in manufacturing and process industries. It solves differential equations which are involved in the design of these types of automatic machinery.

The action of the model in solving these problems is viewed on an oscilloscope which is similar to the picture tube of a television receiver. Movement occurs in the form of a graph which can be easily interpreted and which allows the design of a machine to be changed until it reaches desired performance.

Use of this computer will eliminate former laborious methods of calculating with resultant savings of time and money to the user. It is uniquely flexible in that it can be used with heavy or light machinery of any size.

The Institute's Irving Glassman was project engineer of the development and was assisted by John Bauer and Robert Aaron, research engineers.

Equipment Measures Roll Temperature without Touching Surface of Roll.—A new Roll Surface Temperature Unit developed by Leeds & Northrup Co. measures temperature of a moving roll surface without touching the roll. Because of unique design, it cannot scratch, score, or in any way destroy delicate surface films. Temperature detected by the Unit is recorded automatically by either a Micromax or Speedomax instrument supplied as an integral part of the complete equipment. The recorder can be equipped to operate signals or controls.

Applicable to roll diameters down to 9 in. and to flat surfaces, the new detecting unit operates independent of surface speed, emission characteristics, and finish. It can be mounted at the center of the roll or at any other location, wherever actual temperature measurement is desired.

This new primary element works on the principle that a moving object carries with it a thin, closely adhering layer of air nearly the same temperature as the moving surface. Mounted $\frac{1}{32}$nd of an inch from the surface is a measuring head, contoured to fit the roll. A continuous stream of air is sucked past

the measuring element and thus heats it to virtually the same temperature as the roll surface.

Although recorded temperature is a few degrees lower than true surface temperature, the difference remains substantially constant and is therefore negligible in process control (−4° F. or less). The Unit provides temperature measurements well within limits of industrial requirements. Air suction rate and surface speed are non-critical over wide limits. Measurements are not appreciably affected by ambient temperature.

Compound joint with spacing slide makes it easy to position the measuring head concentric with the roll and at proper spacing. Should material adhere to the roll surface, a safety bumper retracts the measuring head. The Unit can also be supplied with a safety switch which operates in conjunction with this retracting device. This safety actuates a bell, horn, or light to signal the operator and freezes the control until the measuring head is reset.

Pliotherm.—Pliotherm, a revolutionary new rubber compound manufactured by The Goodyear Tire & Rubber Company, is an elastomeric material which by proper compounding is capable of conducting electricity either for transformation into thermal energy or for the dissipation of electrical charges. Some years ago Goodyear Research men discovered that rubber could be made electrically conductive by a special pigmentation process. The conductance, however, was of a very low magnitude. During the last war, constant experimenting to improve and modify the technique resulted in an electrically conducting rubber with a high order of conductance. Desirable characteristics such as good flexibility, high tensile strength, and good processability were preserved in this new material.

Although used commercially to some extent, Pliotherm must still be considered in a pioneering stage, since new applications are being conceived every day. Some of the suggested applications include: radiant panel heating in mild-climate homes, in summer cottages, and in bus, airplane or railway car heating; de-icer treads for exterior building steps and stairways and on transportation facilities; sheathing for water pipes exposed to possible freezing temperatures; heating pads; heated gaskets for the quick release of covers, doors, etc., that tend to freeze when moisture is present; and a variety of industrial uses including heated rubber rolls for laminating work, extruded resistance wires and contact switches for the operation of relays and counters.

Pliotherm becomes a heating element simply by attaching suitable electric connections to it. The sheet is sandwiched between electrically insulated cover plies of rubber or rubberized fabric and the entire mass is bonded together in the manner of conventional "plied up" rubber articles. Any of the wide range of colors possible in rubber compounding can of course be used in the insulating outside rubber plies, but for practical purposes best results are obtained by using black, dark brown, red or green. The Pliotherm heater lends itself well to decorative and functional design. It can be easily combined with wall board, wood panels, rubber and plastic wall and floor coverings, moulding, and the like.

A new hand operated map reproduction device capable of making 200 copies of four color maps in sizes up to 22 × 29 inches has been developed by

the Engineer Research and Development Laboratories, Ft. Belvoir. The duplicator uses alcohol as the dampening medium instead of water, operates under temperatures from plus 25° to plus 130° F. Typing, writing, or drawing is reproduced from a master sheet backed up by a special carbon paper. Design makes maxium use of non-strategic materials, produces a final weight of 468 pounds for machine, accessory equipment, supplies and carrying chests which makes the duplicator transportable by air. The machine can be used commercially in preparing large charts and forms.

Rare Earth in California.—Valuable deposits of rare earth minerals newly discovered in southern California by Geological Survey scientists may enable the United States to become largely self sufficient in those elements. The discovery is of the highest importance, because the United States is at present dependent upon foreign sources for its supply.

Intensive studies of the area in San Bernardino County were carried out by the Geological Survey. These studies indicate that an area roughly 6 miles long and 2 miles wide contains numerous deposits of rare earths, chiefly cerium, lanthanum, neodymium, and praseodymium. The full size and extent of these deposits are still not known, but some of them appear to be very large. D. Foster Hewett, geologist in charge of the field investigations for the Geological Survey, estimates that a single deposit in the area, if it extends to a depth of 100 feet, may contain about 50,000 tons of rare earth bearing minerals.

The rare earths are known most popularly as "flints" used in lamps and pocket lighters, but more important uses are found in such items as arc lamps, tracer bullets, and luminescent shells. Metallurgists have found rare earths of considerable value in alloys of light metals and for improving the quality of some steel. Various chemical compounds of rare earths are used in many industries. Recently it has been found that several of the rare earths are excellent absorbers of slow neutrons, and thus will find important uses in atomic research.

Most important of the new discoveries are several bodies of barite-bastnaesite rock that appear, from representative samples, to contain from 10 to 40 per cent of cerium earths. These ore bodies occur within an area of barite-carbonate rock, approximately 20 acres in extent. Samples from the largest high-grade deposit, which crops out in an area of approximately 90 by 130 feet, show an average rare earth content of about 20 per cent.

Although the rare earth content alone makes the discovery spectacular, barite is also a valuable constituent of the deposits. Barite is widely used by the petroleum industry as a constituent of heavy drilling muds, and it has many other important industrial and chemical uses. If the large barite-carbonate rock area extends to a depth of 100 feet, it should contain some hundreds of thousands of tons of barite.

The rare earths now are obtained from the mineral monazite, a phosphate of the cerium earths. Prior to about 1910, this country produced enough monazite from the stream gravels of the Carolinas to satisfy the then small world needs. The discovery of richer beach deposits of monazite in foreign countries, chiefly Brazil and India, forced domestic mines to close shortly thereafter. Recently developed uses for rare earth elements have increased the national requirements faster than it has been possible to increase imports.